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UNIFORM POWER DISTRIBUTION IN A REFLECTED HOMOGENEOUS REACTOR

Lloyd N. Hoover
and
Robert W. Kennedy

UNIFORM POWER DISTRIBUTION IN A REFLECTED HOMOGENEOUS REACTOR

by

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(1955)

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE IN NUCLEAR ENGINEERING
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Cambridge 39, Massachusetts

August, 1956

Professor Leicester F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

The attached thesis entitled "Uniform Power Distribution in a Reflected Homogeneous Reactor" is submitted in partial fulfillment of the requirements for the degree of Master of Science in Nuclear Engineering.

Respectfully,

ACKNOWLEDGEMENTS

The authors wish to express their gratitude to Professors Thomas H. Pigford and Melville Clark, Jr., for their supervision and helpful suggestions in completing this thesis. The authors also wish to express their appreciation for the help given by members of the M. I. T. Digital Computer staff.

UNIFORM POWER DISTRIBUTION IN A REFLECTED HOMOGENEOUS REACTOR

by

Lloyd N. Hoover, Lieutenant, U. S. Navy

and

Robert W. Kennedy, Lieutenant, U. S. Navy

Submitted to the Department of Chemical Engineering in August, 1956, in partial fulfillment of the requirements for the degree of Master of Science in Nuclear Engineering.

ABSTRACT

The spatial distribution of power in a nuclear reactor tends to follow the neutron distribution because the energy resulting from nuclear fission is released largely at the point where neutrons induce fission. Leakage in an unreflected reactor causes both distributions to be higher near the reactor center. The power at the center is limited by the maximum temperature which can be tolerated by core materials with the result that outer regions operate at less than ultimate capacity. Ideally all regions should be operated at the maximum tolerable temperature, which would require uniform spatial distribution of power. Thus an important improvement in a reactor's power output can be achieved by making the power distribution more nearly uniform.

One simple, economical method of achieving uniform power consists of compensating for the decrease in power near the edge of the core by means of a reflector. If the return of neutrons from the reflector to the core can be adjusted to compensate for the leakage factor, the power distribution can be markedly improved. This method, applied to a reflected spherical homogeneous reactor, is studied in this paper.

The distribution of neutrons in a reactor is specified reasonably well by the diffusion type equations to be discussed in Chapters I and II. Multi-group methods of solution are used because of the importance of the distribution of fast neutrons. Assuming two groups of neutrons was considered inadequate for the intermediate type of re-reactors which are studied. Multi-group calculations are so time-consuming that the use of computing machines is essential. Chapters I and II develop techniques for use with M. I. T.'s digital computer Whirlwind I.

ABSTRACT
(continued)

To simplify the problem, a basic reactor is selected with highly enriched fuel, sodium coolant, beryllium moderator and reflector. The reactor is assumed to be homogeneous and in the steady state. The effect of reactor poisoning is not considered. Spherical geometry is assumed.

It was found that by suitable adjustment of the material proportions in both core and reflector the average power could be raised to within 16% of the maximum power. Results are presented and discussed in Chapter III.

Thesis Supervisors: Thomas H. Pigford
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UNIFORM POWER DISTRIBUTION IN A
REFLECTED HOMOGENEOUS REACTOR

INTRODUCTION

Neutron leakage from an unreflected reactor results in a lower concentration of neutrons toward the reactor edge. By virtue of the fact that the power resulting from nuclear fission induced by neutrons is largely liberated at the point where the fission occurs, the spatial distribution of power also is depressed near the reactor edge. This imposes a limitation on a reactor's power output. The maximum power density at any point is limited by temperature considerations. Outer regions, with lower power density, operate at temperatures lower than their ultimate capacity. Elementary considerations indicate that the average power in an unreflected spherical reactor is only one third of the maximum power produced at the reactor center. Thus, if a uniform power distribution could be achieved, an important increase in reactor output could be realized.

Spatial variations of the proportions of the materials in the core, together with use of a reflector, is one method of improving power distribution. Another is the use of a poison to depress the neutron concentration near the reactor center. These methods have the disadvantages of increased complexity and cost. A simple, economical alternative, the use of neutrons reflected back into the core from the reflector to compensate for the power depression in the outer core, is investigated here.

The diffusion equations to be discussed in Chapters I and II specify reasonably well the distribution of neutrons in a reactor. Since neutron energy varies widely and since the coefficients of the terms in the diffusion equation depend on neutron energy, some simplification is necessary. One method, applied to thermal reactors, assumes that neutrons are slowed to thermal energies immediately after birth. The coefficients of the terms in the diffusion equation are then determined at thermal energies, and a simple analytical solution is possible.

However, in this problem, the number of neutrons with energies greater than thermal must be increased in order to supply the reflector with fast neutrons for moderation and reflection back into the core to compensate for leakage. Therefore, the distribution of higher energy neutrons must be known, and one-group calculations are inadequate.

Considering two groups of neutrons--fast and slow--would give some representation of fast neutrons which leak from the core to the reflector, where they may be moderated and returned to the core as slow neutrons. However, better solutions are possible if more than two groups are considered.

Multi-group calculations require solutions of diffusion equations for each energy group of neutrons considered and are very laborious and time-consuming. The use of a high speed digital computer serves to minimize both of these problems. In Chapters I and II, diffusion equations for

four and six energy groups respectively are broken down into a set of simple calculations which a computer can perform to solve the equations. The actual programmed set of instructions is used with M. I. T.'s digital computer, Whirlwind I. The basic method is applicable to other digital computers by a suitable transformation of the specific set of instructions. Whirlwind I requires less than a minute to solve the equations, but a period of from two to three minutes is required for the computer to transform the input of decimal numbers and interpreted instructions to its own binary system. This conversion time can be reduced by rewriting the program in the basic Whirlwind code. The Whirlwind I computer will perform this function automatically and will cut a new tape containing the converted program.

In this paper a basic reactor composition and configuration are chosen, and the proportions of the various materials of the core and reflector varied in order to determine their effect on the power distribution. The fundamental reactor type studied was one considered suitable for high output mobile reactor application.

The requirements of this type of reactor led to the use of core materials capable of withstanding high temperatures. A liquid metal coolant was chosen in order to avoid having to pressurize the core. Of the liquid metals available, sodium was chosen because of its superior heat

removal characteristics and because of its compatibility with other materials at high temperature with respect to corrosion.

Once sodium was chosen as the coolant, stainless steel became the best material for cladding in view of its high temperature strength and resistance to corrosion by sodium.

A dispersal fuel element of uranium dioxide in a matrix of stainless steel was chosen to meet high temperature requirements and to minimize radiation damage. Calculations for a fuel element of this type have been made.*

Beryllium was chosen as a moderator and coolant because of its high moderating ratio and good thermal properties.

The average operating temperature of the reactor was taken as 1000°F, which corresponds to a thermal neutron energy of 0.07 ev.

For mathematical simplification, spherical geometry was assumed. The reflector thickness was uniform. Both core and reflector were assumed to be homogeneous and in the steady state. The effect of reactor poisons was not considered. Thus an actual reactor would require additional reactivity to overcome the effects of fission product poisons.

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*"Proceedings of the International Conference on the Peaceful Uses of Atomic Energy," Volume 9, page 561, C. E. Weber and H. H. Hirsch.

CHAPTER I

Method of Finite Differences

L. N. Hoover

This chapter will develop a program for solving multigroup neutron diffusion problems in spherical geometry on the Whirlwind I (WWI) digital computer using a finite difference approximation to the age diffusion equation. The solution will give the power distribution, the neutron flux distribution in each of several energy groups, and the critical radius for a reflected spherical nuclear reactor having a homogeneous core and a homogeneous reflector. The program will be developed for neutrons in four energy groups. By suitable modification of the basic program, the number of energy groups and the number of regions considered can be increased. Four energy groups represent the maximum number of groups which this program will conveniently handle using only the internal storage of WWI. To increase the number of energy groups considered will require using auxiliary storage, which has the disadvantage of increasing the machine time required for a solution.[1] The four-group program requires approximately three minutes of computer time for a complete solution. For a parameter study, the use of a larger number of energy groups was not considered to be advantageous, since very little improvement in results would be achieved by the use of more groups.

I. Age Diffusion Equation

The age diffusion equation provides a good description of the neutron behavior of a large class of nonthermal reactors. The following development is concerned with reactor assemblies that are large compared with the neutron mean free path, and in which the neutron energy is reduced primarily by elastic collisions. The method accounts for both fast and thermal fission, for the fission spectrum, and for inelastic scattering.

The basic equation used is the age-diffusion, which gives good results for the case of weak capture and a cross section that varies slowly with energy,[2]

$$-D\nabla^2\phi(u) + \Sigma_a(u)\phi(u) = -\frac{\partial q(u)}{\partial u} + S(\underline{r},u) \qquad (1.1)$$

The term $-\frac{\partial q(u)}{\partial u}$ represents the excess neutrons entering the lethargy interval over those leaving, assuming that the slowing down density, q, can be treated as a continuous function of the lethargy, u. This excess number of neutrons is balanced in the steady state by the leakage of neutrons out of the region on interest, which is $-D\nabla^2\phi(u)$, where $\phi(u)$ is the flux per unit lethargy interval, and by the neutron absorption, $\Sigma_a(u)\ \phi(u)$. The term $S(\underline{r},u)$ represents neutrons entering lethargy u at a space point $\underline{r}$ as a result of production by fissions, plus inelastic scattering from a higher energy.

There are advantages to expressing Equation (1.1) in terms of the slowing down density, q, rather than the

flux, ϕ. The relationship between the flux per unit lethargy interval and the slowing down density in the case of slowly varying capture cross section and weak absorption is[2]

$$\phi(u) = \frac{q(u)}{\xi \Sigma_s(u)} \tag{1.2}$$

For energies below the source energy, the same number of neutrons must slow down past each energy, except for the effects of leakage and absorption. Hence, for reactors with small absorption in the slowing down range, the slowing down density tends to be a less sensitive function of energy than the flux, ϕ, which will vary more rapidly than q as $\Sigma_s(u)$ varies.

In terms of q, Equation (1.1) becomes

$$-D\nabla^2\left[\frac{q(u)}{\xi\Sigma_s(u)}\right] + \frac{\Sigma_a(u)}{\xi\Sigma_s(u)}\,q(u) = -\frac{\partial q(u)}{\partial u} + S(\underline{r},u) \tag{1.3}$$

In Equation (1.3), u is a continuous variable. For multigroup calculations, Equation (1.3) is integrated over the lethargy range from u_1 to u_2, corresponding to the width of the i'th energy group. $\Sigma_s(u)$ is considered as independent of position and brought in front of the Laplacian operator, ∇^2. The age-diffusion equation for lethargy group i then becomes[4]

$$-U_i\left(\frac{D}{\xi\Sigma_s}\nabla^2 q\right)_{i\ \mathrm{avg}} + U_i\left(\frac{\Sigma_a}{\xi\Sigma_s}\,q\right)_{i\ \mathrm{avg}} = -\,q_i^{\mathrm{OUT}}$$

$$+\ q_i^{\mathrm{IN}} + \int_{U_i} S(\underline{r}_1 u)\,du, \tag{1.4}$$

where $U_i = \int_{u_1}^{u_2} du$. The averages are with respect to u in the group. q_i^{IN} is the degradation of neutrons into the i'th group from the group of next higher energy, and q_i^{OUT} is the degradation out of the i'th group. The approximation of replacing the averages of products appearing in Equation(1.4) by the product of averages yields

$$-\left(\frac{UD}{\xi\Sigma_s}\right)_{i\ avg} \nabla^2 (q)_{i\ avg} + \left(\frac{U\Sigma_a}{\xi\Sigma_s}\right)_{i\ avg} (q)_{i\ avg}$$

$$+ q_i^{OUT} = q_i^{IN} + \int_{U_i} S(\underline{r},u)du \qquad (1.5)$$

This procedure is justified if either q or the cross sections do not change radically over the group.

To solve Equation (1.5) when q_i^{IN} and S are known, it is necessary to postulate an additional relationship between $(q)_{i\ avg}$, q_i^{IN}, and q_i^{OUT}. The simplest method would be to assume a linear relationship for q across the lethargy group and to take an arithmetic average. However, with only four lethargy groups, the large range of lethargy per group tends to make the linear relationship inaccurate. A suitable method for use with a small number of energy groups is to use the relationship

$$(q)_{i\ avg} = fq_i^{OUT} \qquad (1.6)$$

where f is a number chosen by trial and error for each group and region of the assembly so that f times the spatial average of q_i^{OUT} gives a correct value for the spatial average of $(q)_{i\ avg}$. The solution is not sensitive to the choices of f, and it is usually possible, on the basis of

experience, to make adequate choices the first time. From the problem solution, the values of $(q)_{i\ avg}$ are plotted versus lethargy as in Figure 12. A smooth curve is faired through these values of $(q)_{i\ avg}$ to represent the actual process of continuous slowing down. From this curve, refined values of f can be determined and used to improve the solution, if the initial estimate was greatly in error.

II. Average Nuclear Properties of Material in a Lethargy Group

Equation (1.5) expresses the age diffusion equation for lethargy group i in terms of material properties averaged over the i'th group. Within each group, the neutrons are assumed to diffuse without energy loss until they have suffered the average number of collisions which would be required to decrease their energy to that of the next lower group. At this point it is supposed that the neutrons are suddenly transferred to the latter group. This process is supposed to continue while the neutron energy is degraded from the group of birth energy to that of lowest (thermal) energy.

Since the energy spectrum of the neutrons in a reactor covers a continuous range from thermal energies to about 10 Mev, the group-diffusion concept is evidently an approximation. This is partially overcome by assigning appropriate average values for each group to the various properties, such as the cross section and the diffusion

coefficient of the core and the reflector.

a. Diffusion Coefficient. The average diffusion coefficient, D_i, for energy group i is derived from a consideration of the fast neutron current density. The definition of D_i is[3]

$$D_i = \frac{\int_{E_2}^{E_1} D(E)\,\nabla \phi(\underline{r},E)\,dE}{\int_{E_2}^{E_1} \nabla \phi(\underline{r},E)\,dE} \qquad (2.1)$$

It can be seen from this equation that D_i has the same value at different spatial points within the region being considered only if $\phi(\underline{r},E)$ is separable into a function of $\underline{r}$ multiplied by a function of E, i.e., if the variation in the neutron flux spectrum from point to point in the medium can be neglected. This assumption is usually made for simplicity. Hence, with D(E) replaced by $\frac{1}{3\Sigma_{tr}(E)}$, the expression for D_i becomes

$$D_i = \frac{\int_{E_2}^{E_1} \frac{1}{3\Sigma_{tr}(E)}\,\phi(E)\,dE}{\int_{E_2}^{E_1} \phi(E)\,dE} \qquad (2.2)$$

For the asymptotic case where $E \ll \alpha E_0$, which is assumed to apply for all energy groups considered here using Be as the moderator, and for the case of weak absorption

$$\phi(E) = \frac{q(E)}{E\xi\Sigma_s(E)} \quad \text{or} \quad \phi(u) = \frac{q(u)}{\xi\Sigma_s(u)}$$

In lethargy notation we find that Equation (2.2) becomes

$$D_i = \frac{\displaystyle\int_{u_1}^{u_2} \frac{q(u)\,du}{3\xi\Sigma_{tr}(u)\Sigma_s(u)}}{\displaystyle\int_{u_1}^{u_2} \frac{q(u)\,du}{\xi\Sigma_s(u)}}$$

$$= \frac{U_i\left(\frac{q}{3\xi\Sigma_{tr}\Sigma_s}\right)_{i\ avg}}{U_i\left(\frac{q}{\xi\Sigma_s}\right)_{i\ avg}} = \frac{\left(\frac{1}{3\xi\Sigma_{tr}\Sigma_s}\right)_{i\ avg} q_{i\ avg}}{\left(\frac{1}{\xi\Sigma_s}\right)_{i\ avg} q_{i\ avg}}$$

The last step involves the approximation of replacing the averages of products by the product of averages. This is justified if either q or the cross sections do not change radically over the group. Hence, this is a criterion which must be considered in choosing the energy subdivisions of the group. Thus the average diffusion coefficient for group i is given by

$$D_i = \frac{\left(\frac{1}{3\xi\Sigma_s\Sigma_{tr}}\right)_{i\ avg}}{\left(\frac{1}{\xi\Sigma_s}\right)_{i\ avg}} \qquad (2.3)$$

b. Total Flux Per Energy Group. The total flux in energy group i, NV_i, is defined as

$$NV_i = \int_{u_1}^{u_2} \phi(u)\,du = \int_{U_i} \frac{q(u)\,du}{\xi\Sigma_s(u)}$$

Use of the same approximations as for the diffusion coefficient gives the result, where $U_i = \int_{u_1}^{u_2} du$

$$NV_i = U_i\left(\frac{1}{\xi\Sigma_s}\right)_{i\ avg} (q)_{i\ avg} \qquad (2.4)$$

or

$$(q)_{i\ avg} = \frac{NV_i}{U_i\left(\frac{1}{\xi \Sigma_s}\right)_{i\ avg}} \tag{2.4a}$$

c. Average Absorption Cross Section Per Group.

The average absorption cross section for energy group i is defined as

$$\Sigma_{a_i} = \frac{\int_{u_1}^{u_2} \Sigma_a(u)\phi(u)du}{\int_{u_1}^{u_2} \phi(u)du} = \frac{\int_{u_1}^{u_2} \frac{\Sigma_a(u)q(u)du}{\xi \Sigma_s(u)}}{\int_{u_1}^{u_2} \frac{q(u)du}{\xi \Sigma_s(u)}}$$

When the same approximations as before are made, the average absorption cross section for energy group i becomes

$$\Sigma_{a_i} = \frac{(\Sigma_a/\xi \Sigma_s)_{i\ avg}}{(1/\xi \Sigma_s)_{i\ avg}} \tag{2.5}$$

III. Spatial Integration of the Group Diffusion Equations.

Equation (1.5) describes the neutron behavior in energy group i. Substitution in (1.5) of (2.5) plus the following relationships developed previously

$$q_i^{OUT} = \frac{(q)_{i\ avg}}{f} \tag{1.6}$$

$$D_i = \frac{\left(\frac{1}{3\xi \Sigma_{s\ tr}}\right)_{i\ avg}}{\left(\frac{1}{\xi \Sigma_s}\right)_{i\ avg}} \tag{2.3}$$

$$(q)_{i\ avg} = \frac{NV_i}{U_i\left(\frac{1}{\xi \Sigma_s}\right)_{i\ avg}} \tag{2.4a}$$

gives

$$-D_i \nabla^2 NV_i + \left\{ \Sigma a_i + \frac{1}{f_i U_i \left(\frac{1}{\xi \Sigma_s}\right)_{i\ \mathrm{avg}}} \right\} NV_i$$

$$= q_{i-1}^{\mathrm{OUT}} + \int_{U_i} S(\underline{r},u)\,du \qquad (3.1)$$

For spherical geometry, the following substitutions will simplify (3.1) by eliminating the Laplacian operator ∇^2:

$$ {}^iF_n = r_n NV_i \qquad (3.2a)$$

$$ {}^iQ_n^{\mathrm{OUT}} = r_n q_i^{\mathrm{OUT}} \qquad (3.2b)$$

$$ {}^iS_n = r_n \int_{U_i} S(r_n,u)\,du \qquad (3.2c)$$

A new notation is adopted. Let i refer to the i'th energy group as previously and the subscript n refer to the n'th space point located at a distance r_n from the origin of the spherical coordinates. As before, the source term ${}^iS_n$ represents neutrons entering group i at radius r_n as a result of inelastic scattering and fissions in all groups. Use (3.2) in (3.1) to get

$$-D_i \frac{d^2\,{}^iF_n}{dr^2} + \left[\Sigma a_i + \frac{1}{f_i U_i \left(\frac{1}{\xi \Sigma_s}\right)_{i\ \mathrm{avg}}} \right] {}^iF_n = {}^{i-1}Q_n^{\mathrm{OUT}} + {}^iS_n \qquad (3.3)$$

The space points are chosen to divide the spatial region into a set of subintervals of width Δr. The finite difference approximation

$$\left(\frac{d^2\,{}^iF_n}{dr^2}\right)_{r_n} = \frac{{}^iF_{n+1} + {}^iF_{n-1} - 2\,{}^iF_n}{(\Delta r)^2} \qquad (3.4)$$

and Equation (3.3) gives the recurrence relation

$$ {}^{i}F_{n+1} = k_i \, {}^{i}F_n - {}^{i}F_{n-1} - {}^{i}I_n \tag{3.5} $$

where

$$ k_i = 2 + \frac{b_i (\Delta r)^2}{D_i} \tag{3.5a} $$

$$ b_i = \Sigma_{a_i} + \frac{1}{f_i U_i \left(\frac{1}{\xi \Sigma_s} \right)_{i\ \text{avg}}} \tag{3.5b} $$

$$ {}^{i}I_n = \frac{(\Delta r)^2}{D_i} \left[{}^{i-1}Q_n^{\text{OUT}} + {}^{i}S_n \right] \tag{3.5c} $$

Equation (3.5) can be verified by direct substitution of Equation (3.4) into Equation (3.3). The above substitutions follow a method developed by Ehrlich and Hurwitz.[4]

a. Boundary Conditions. The boundary conditions for the numerical solution of Equation (3.5) are that ${}^{i}F_n = 0$ at $r = 0$ and also at the extrapolated end point outside the assembly. Another condition is that the neutron flux and current density must be made continuous at boundaries between regions of different material properties. Suitable choices of Δr must be made for each region so that the boundaries will occur at space points. At a boundary point, M, located at radius $r = r_M$ between two different media, these boundary conditions give (primes refer to the outer region)

${}^{i}F_M = {}^{i}F'_M$, for continuity of flux,

$J_M = J'_M$, for continuity of current density,

where

$$J = -D\,\frac{d(NV)}{dr} = -\frac{D}{r^2}\left(-F + r\,\frac{dF}{dr}\right)$$

It can be shown that (see Appendix C)

$$\left(-F + r\,\frac{dF}{dr}\right)\Big|^{r_M} = \frac{r_M}{2\Delta r}\left(F_{M+\Delta r} - F_{M-\Delta r}\right) - \frac{2}{3}F_M \qquad (3.5d)$$

$$-\frac{1}{6}\left(F_{M+\Delta r} + F_{M-\Delta r}\right)$$

Therefore, the boundary condition for neutron current density, $J = J'$, is

$$\frac{2}{3}F_M + \frac{1}{6}\left(F_{M+\Delta r} + F_{M-\Delta r}\right) - \frac{r_M}{2\Delta r}\left(F_{M+\Delta r} - F_{M-\Delta r}\right)$$

$$= \frac{D'}{D}\Big[\frac{2}{3}F'_M + \frac{1}{6}\left(F'_{M+\Delta r'} + F'_{M-\Delta r'}\right) - \frac{r_M}{2\Delta r'}\cdot$$

$$\left(F'_{M+\Delta r} - F_{M-\Delta r}\right)\Big] \qquad (3.6)$$

b. Method of Solution. The recurrence relation for ${}^{i}F_n$, Equation (3.5), will be solved using a procedure suggested by Ehrlich and Hurwitz.[4] This procedure is particularly convenient for use with the WWI digital computer.

To solve Equation (3.5) subject to the above boundary conditions, new variables α_n and β_n will be introduced. These variables are related to the homogeneous ($I_n = 0$) solution of (3.5), A_n, by the defining relationships

$$\alpha_{n+1} = \frac{A_{n+1}}{A_n} \text{ and } \beta_n = \sum_{j=0}^{n} A_j\,\frac{I_j}{A_n}$$

When the above expression for α is substituted into (3.5), with $I_n = 0$, it is found that

$$\alpha_{n+1} = k - \frac{1}{\alpha_n} \qquad (3.7)$$

The recurrence relationship for the β's is

$$\beta_n = \frac{\beta_{n-1}}{\alpha_n} + I_n \qquad (3.8)$$

which can be verified by direct substitution of the definition of β_n into (3.8). A combination of Equations (3.5), (3.7), and (3.8) gives the following relationship

$$F_n = \frac{F_{n+1} + \beta_n}{\alpha_{n+1}} \qquad (3.9)$$

This is proved as follows:

Substitute (3.7) and (3.8) for α and β into (3.9) and combine terms. This gives

$$\frac{1}{\alpha_n}\left[F_n + \beta_{n-1}\right] = F_n k - F_{n+1} - I_n$$

By Equation (3.5), the right side becomes F_{n-1}. This is then Equation (3.9), when n is replaced by n+1.

Equation (3.7) is used to solve for the α's, starting with $\alpha_2 = k$, since $A_o = 0$ and therefore, from the definition of α, $\alpha_1 = \infty$. The next step is to use (3.8) to solve for the β's, starting with $\beta_1 = I_1$. Finally (3.9) is solved for the F's from the outside to the center of the spherical reactor assembly, starting with $F_n = 0$ at the extrapolated end point.

The equations for α and β have the same mathematical form but different constants in regions of different material properties. At the boundary between regions, α and β are single valued in order to maintain continuity of flux. Thus Equation (3.9) is unchanged at a boundary. At the

boundary point M, at radius r_M, separating two different media, the boundary condition for continuity of current density can be satisfied by using Equation (3.6). The substitution of the recurrence relationships for α and β into (3.6) leads to the following relationships across boundary point M (primes refer to the outer media):

$$\alpha'_{M+1} = K - \frac{\Delta r'}{s\Delta r}\cdot\frac{1}{\alpha_M} \tag{3.10}$$

$$\beta'_M = pI_M + \left(\frac{1}{2} + \frac{\Delta r'}{6r_m}\right)I'_M + \frac{\Delta r'}{s\Delta r}\cdot\frac{\beta_{M-1}}{\alpha_M} \tag{3.11}$$

where

$$s = \frac{D'}{D}$$

$$p = \frac{1}{2s}\left(\frac{\Delta r'}{\Delta r} - \frac{\Delta r'}{3r_M}\right) = \frac{22}{3}\,\frac{\Delta r'}{sR}\ {}^{*} \tag{3.12}$$

$$K = pk + \left(1 + \frac{\Delta r'}{3r_M}\right)\frac{k'}{2} + \left(1 - \frac{1}{s}\right)\frac{2\Delta r'}{3r_M}$$

c. Thermal Group. The thermal neutrons are assumed to be in a group of essentially constant lethargy. It is assumed that neutrons enter the thermal group only by elastic scattering from the group of next higher energy. The thermal group is treated the same as other groups except for changing the definition of NV_T (where the subscript T stands for the thermal group). For the thermal group

---

* for the case considered here with 15 space points in the core so that $\Delta r = R/15$, where R is the core radius.

$NV_T = \phi_T$

d. Energy Groups. Energy Group I is chosen from 10 Mev to 0.1 Mev ($u = 0$ to $u = 4.606$). Glasstone[5] plots the fission neutron energy spectrum, and it can be seen from this plot that very little error is introduced by assuming that all neutrons released in fission are born into Group I. Groups II and III are chosen to give approximately equal lethargy widths down to a thermal energy corresponding to 1000°F.

The energy groups will be as follows:

Group	Lethargy, u	Energy, E	Lethargy Width
I	0 - 4.606	10-0.1 Mev	4.606
II	4.606 - 10.81	100-0.2 Kev	6.204
III	10.81 - 18.80	200-0.070 ev	7.990
T	18.80	0.070 ev (1000°F)	0

e. Inelastic Scattering. Provision is made for considering inelastic scattering into an energy group by the definition of the term $\int_{U_i} S(\underline{r},u)du$ in the age diffusion equation. This term is defined to be composed of fission neutrons born into group i plus those inelastically scattered into group i from higher energies. Since a neutron must possess a minimum of about 0.1 Mev of energy if it is to be involved in an inelastic scattering process,[6] inelastic scattering occurs only in Group I. The group widths are large so there will be no inelastic scattering across groups. This can be seen from plots of the energy

distribution of inelastically scattered neutrons as given in the Reactor Handbook, Reference (7).

IV. Programming for the Whirlwind I Computer.

Whirlwind I (WWI) is a digital computer capable of performing simple mathematical operations (addition, subtraction, multiplication, and division) with extreme rapidity under the control of a set of instructions, called the program, which is stored in the computer. A detailed account of WWI computer operation is given in Chapter II, Section II, of this thesis. For additional information, the reader is referred to Reference (1), which completely covers WWI operating techniques.

a. Numerical Operations. Section III of this chapter develops the relationships which are used in programming the WWI solution. The steps of the solution are outlined below:

(1) Start by assuming the spatial distribution of fission neutrons born into Group I as a result of fissions in all groups. (This assumed distribution will be improved to the correct value by successive iterations.)

(2) From the assumed spatial distribution of fission neutrons into Group I, which is $\int_{U_I} S(r_n, u)\,du$, use (3.2c) to get ${}^{I}s_n$

$$ {}^{I}s_n = r_n \int_{U_I} S(r_n, u)\,du. $$

(3) Calculate the following constants for Group I,

using Equations (3.5) and (3.12):

$$b_i = \Sigma_{ai} + \frac{1}{f_i U_i \left(\frac{1}{\xi \Sigma_s}\right)_{i\ avg}}$$

$$s = \frac{D'}{D}$$

$$p = \frac{1}{2s}\left(\frac{\Delta r'}{\Delta r} - \frac{\Delta r'}{3r_M}\right)$$

$$k = 2 + \frac{b_i(\Delta r)^2}{D_i}$$

$$K = pk + \left(1 + \frac{\Delta r'}{3r_M}\right)\frac{k'}{2} + \left(1 - \frac{1}{s}\right)\frac{2\Delta r'}{3r_M}$$

(4) Calculate ${}^I\alpha_n$ starting with ${}^I\alpha_2 = k$ and using Equation (3.7), $\alpha_{n+1} = k - \frac{1}{\alpha_n}$. The value of k must be correct for the region considered. At the boundary between core and reflector, where n = 15, boundary condition (3.10) is used, $\alpha'_{16} = K - \frac{\Delta r'}{s\Delta r}\cdot\frac{1}{\alpha_{15}}$.

(5) Calculate ${}^I\beta_n$ starting with ${}^I\beta_1 = {}^I I_1$ and using Equation (3.8), $\beta_n = \frac{\beta_{n-1}}{\alpha_n} + I_n$, where

$${}^I I_n = \frac{(\Delta r)^2}{D_I}\,{}^I S_n \qquad \text{for } 1 \leq n \leq 15$$

$${}^I I'_n = 0 \qquad \text{for } 16 \leq n \leq 25$$

At the boundary, n = 15, Equation (3.11) applies,

$$\beta_{15} = pI_{15} + \left(\frac{1}{2} + \frac{\Delta r'}{6r_{15}}\right)I'_{15} + \frac{\Delta r'}{s\Delta r}\,\frac{\beta_{14}}{\alpha_{15}}$$

(6) Solve for ${}^I F_n$ starting with ${}^I F_{25} = 0$ and working toward the center of the spherical reactor assembly

using Equation (3.9),

$$^{I}F_{n} = \frac{^{I}F_{n+1} + \beta_{n}}{\alpha_{n+1}}$$

(7) Calculate the inelastic scattering, $^{II}S_{n\ in.}$, from Group I into Group II by use of the relationship

$$^{II}S_{n\ in.} = U_{i}\,(x\Sigma_{in.})_{I}\ ^{I}F_{n},$$

where x is the probability that a neutron scattered inelastically in Group I will enter Group II.

(8) Repeat successively steps (2) through (6) for Groups II, III, and T. Note that there are two changes to the procedure used with Group I: First, the input term, $^{II}S_{n}$, is composed only of inelastically scattered neutrons entering Group II (since all fission neutrons are born in Group I), and $^{i}S_{n}$ is zero for Groups III and T; second, the calculation of $^{i}I_{n}$ by Equation (3.5c),

$$^{i}I_{n} = \frac{(\Delta r)^{2}}{D_{i}}\left[\,^{i-1}Q_{n} + {}^{i}S_{n}\right]$$

includes elastic scattering, $^{i-1}Q_{n}^{\ OUT}$, out of the group of next higher energy.

(9) When $^{i}F_{n}$ has been calculated for all energy groups, calculate a new source term, $^{I}S_{n}$, of fission neutrons for Group I from the relationship

$$^{I}S_{n} = \sum_{i=I}^{T} (\nu\Sigma_{f})_{i\ avg}\ ^{i}F_{n},$$

where $(\nu\Sigma_{f})_{i\ avg}$ is the average value in group i. Thus, $^{I}S_{n}$ is proportional to the fission power distribution in the reactor.

(10) The new input term, ${}^{I}S_n$, is normalized and used as the basis for a new iteration, which repeats the above procedure starting with step (3).

b. Iteration Procedure. The iteration procedure has been divided into two steps: (1) relaxing from the assumed input of fission neutrons, ${}^{I}S_n$, to the correct spatial distribution; and (2) changing the radius, R, of the new reactor core to obtain criticality.

Let the superscript m indicate the number of the iteration, with $m = 0$ indicating initially assumed values. Let a circumflex indicate values which have been normalized.

Step (1) corrects the spatial distribution of ${}^{I}S_n$ as follows:

All iterated input terms ${}^{I}S_n{}^{m}$ into Group I, starting from the assumed ${}^{I}S_n{}^{o}$, are normalized with respect to an arbitrarily chosen core spatial point, N, where ${}^{I}S_N{}^{m-1}$ is normalized to unity. A normalizing factor γ is defined such that

$$\gamma = \frac{{}^{I}\hat{S}_N{}^{m-1}}{{}^{I}S_N{}^{m}} = \frac{1}{{}^{I}S_N{}^{m}} \qquad (4.1)$$

and

$${}^{I}\hat{S}_n{}^{m} = \gamma \, {}^{I}S_n{}^{m}$$

where N = the chosen spatial point for normalization to unity

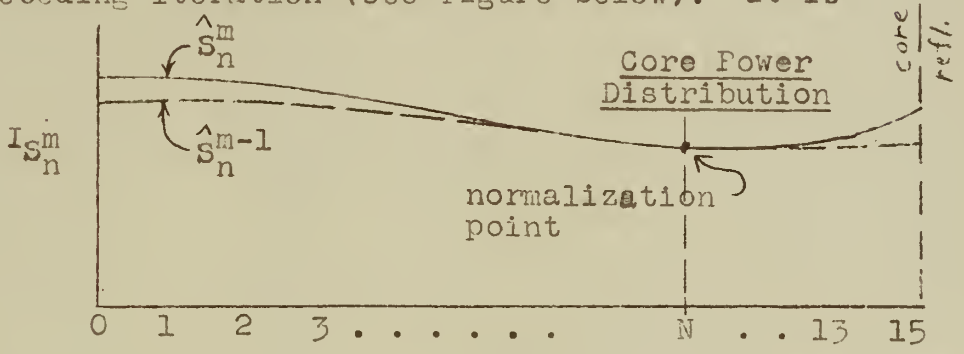
m = the number of the iteration

${}^{I}S_n{}^{m}$ = the calculated input term to Group I at spatial point n

${}^{I}\hat{S}_n^{\,m}$ = normalized source team

${}^{I}\hat{S}_N^{\,m-1} = 1$ = source term of previous iteration normalized to unity at space point N

To determine when to stop iterating, the normalized input term, ${}^{I}\hat{S}_n^{\,m}$, is compared at each spatial point, n, with the preceding iteration (see figure below). It is specified that at each spatial point the ratio $\hat{S}_n^{\,m-1}/\hat{S}_n^{\,m}$ must differ from unity by no more than a small value, ϵ , to permit the program to proceed to the next step. Whenever the value ϵ is exceeded at any space point, the program will reiterate. This is equivalent to requiring that the solution for the core power distribution must have converged sufficiently to permit the solution from one iteration to lie entirely within an envelope determined by ϵ around the solution of the preceding iteration.



The criterion for choosing between iterating or continuing to the next step is based upon the ability of WWI computer to choose one course of action for a positive number and a different one for a negative number (conditional control transfer). Provision is made for the criterion to

operate, irrespective of whether $\hat{S}_n^{m-1}/\hat{S}_n^m$ is greater or less than unity, by the following scheme:

(a) When $\dfrac{\hat{S}_n^{m-1}}{\hat{S}_n^m}$ is greater than unity:

$$1 - \frac{\hat{S}_n^{m-1}}{\hat{S}_n^m} + \epsilon < 0, \text{ not acceptable (reiterate)}$$

$$1 - \frac{\hat{S}_n^{m-1}}{\hat{S}_n^m} + \epsilon > 0, \text{ acceptable (proceed to the next step)}$$

(b) When $\dfrac{\hat{S}_n^{m-1}}{\hat{S}_n^m}$ is less than unity:

$$\frac{\hat{S}_n^{m-1}}{\hat{S}_n^m} - 1 + \epsilon < 0, \text{ not acceptable (reiterate)}$$

$$\frac{\hat{S}_n^{m-1}}{\hat{S}_n^m} - 1 + \epsilon > 0, \text{ acceptable (proceed)}$$

Step (2) of the iteration procedure involves changing the core radius to get criticality. The procedure is as follows:

Step (1) above has required that a plot of the input term $\hat{S}_n^m$ must reach a stable shape before proceding to step (2). In step (2) the last calculated value of γ by (Equation 4.1) is used to test for over or under criticality. A γ less than unity means that the number of fission neutrons entering energy Group I increases from iteration to iteration and, therefore, that the multiplication factor, k,

is greater than unity (and vice versa). For the critical condition, $k = 1$, the shape stabilized source term remains constant for successive iterations, and γ therefore is unity.

If γ is not within a small value ϵ of unity, the program goes through another iteration using a new value of the core radius, R, chosen to come closer to criticality. The new radius for iteration m is

$$R_m = \gamma_{m-1} R_{m-1}$$

Thus, the core radius is increased if $k < 1$, and vice versa. The criteria for determining whether the radius is correct or not is:

(a) When γ is greater than unity:

$\epsilon - \gamma + 1 < 0$ not acceptable (modify radius and reiterate)

$\epsilon - \gamma + 1 > 0$ acceptable (proceed to the next step)

(b) When γ is less than unity:

$\gamma - 1 + \epsilon < 0$ not acceptable

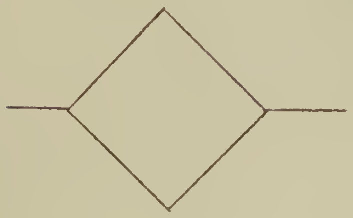
$\gamma - 1 + \epsilon > 0$ acceptable

When the iteration criteria of both step (1) and step (2) are satisfied, the program proceeds to the final step of calculating and printing the core power distribution, the critical radius, and the neutron flux distribution in each energy group.

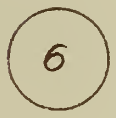
V. Flow Sheet.

The program for the solution of the reactor problem

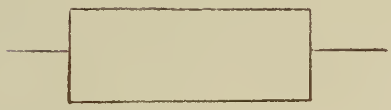
has been described in Section IV. A flow sheet will be developed to show schematically the operations discussed in Section IV. The following conventions will be used:



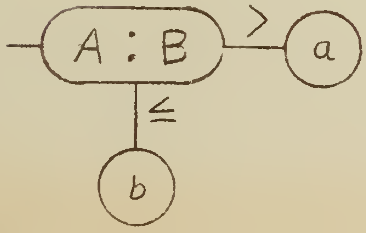
A square box standing on one edge denotes a subroutine for calculating the quantity within the box.



Circles enclosing numbers are used to specify corresponding parts of the flow sheets. The numbers are called "addresses." The circles are called connectors and are put at those places in a flow diagram to which reference will be made at some other point.



A rectangular box denotes that the quantity inside is to be calculated or the operations within are to be performed. Frequently the formula for calculating the quantity is given.



This symbol indicates a conditional transfer to a if A $>$ B and to b if A $\leq$ B.

Subscripts and superscripts will have the following definitions:

i denotes an energy group.

n denotes a space point at radius r_n.

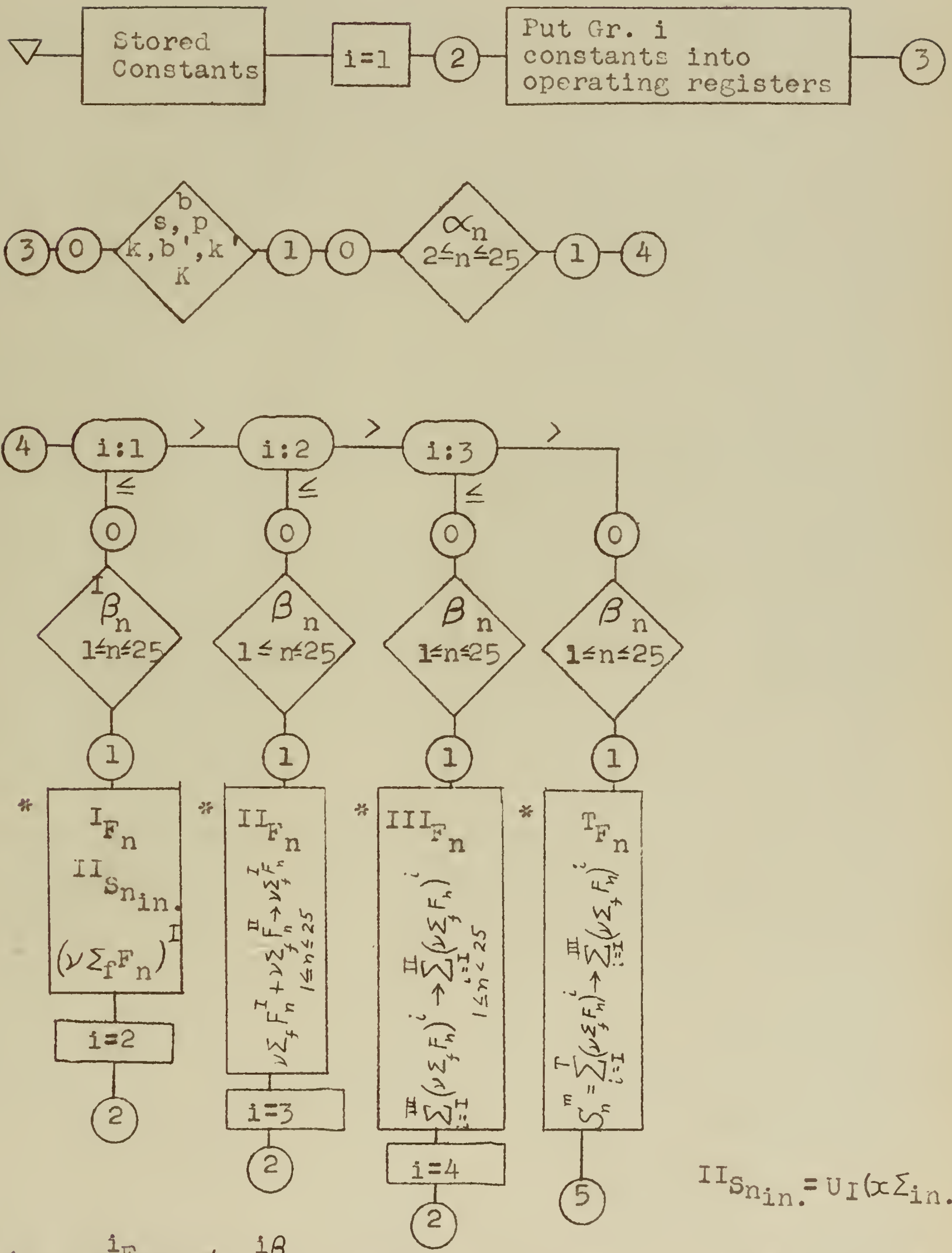
m denotes the number of an iteration, where $m = 0$ denotes values initially assumed.

N denotes an arbitrary space point, $n = N$, chosen as a basis for normalization procedures.

→ the symbol $a \rightarrow b$ means that the quantity a replaces the quantity b in the storage register in which b is stored.

FLOW SHEET FOR FOUR GROUPS

Main Program

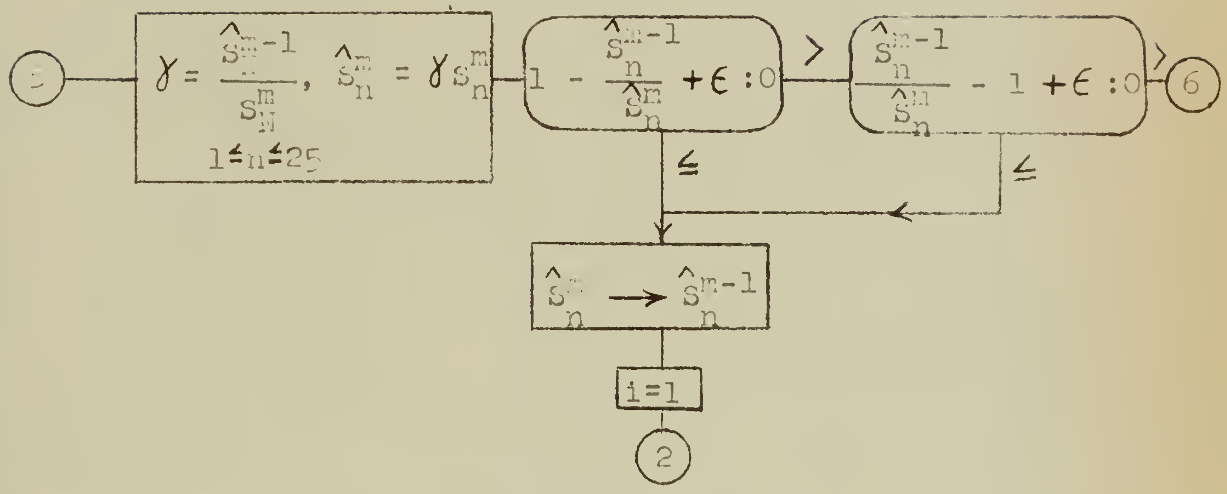

Stored Constants
i=1
2
Put Gr. i constants into operating registers
3
3
0
b s, p k, b', k' K
1
0
α_n $2 \le n \le 25$
1
4
4
i:1
>
i:2
>
i:3
>
≤
≤
≤
0
0
0
0
$^I\beta_n$ $1 \le n \le 25$
β_n $1 \le n \le 25$
β_n $1 \le n \le 25$
β_n $1 \le n \le 25$
1
1
1
1
*
^IF_n
$^{II}S_{n_{in.}}$
$(\nu\Sigma_f F_n)^I$
i=2
2
*
$^{II}F_n$
$\nu\Sigma_f F_n^I + \nu\Sigma_f F_n^{II} \to \nu\Sigma_f F_n^I$
$1 \le n \le 25$
i=3
2
*
$^{III}F_n$
$\sum_{i=I}^{II}(\nu\Sigma_f F_n)^i \to \sum_{i=I}^{III}(\nu\Sigma_f F_n)^i$
$1 \le n \le 25$
i=4
2
*
^TF_n
$S_n^m = \sum_{i=I}^{T}(\nu\Sigma_f F_n)^i \to \sum_{i=I}^{III}(\nu\Sigma_f F_n)^i$
5

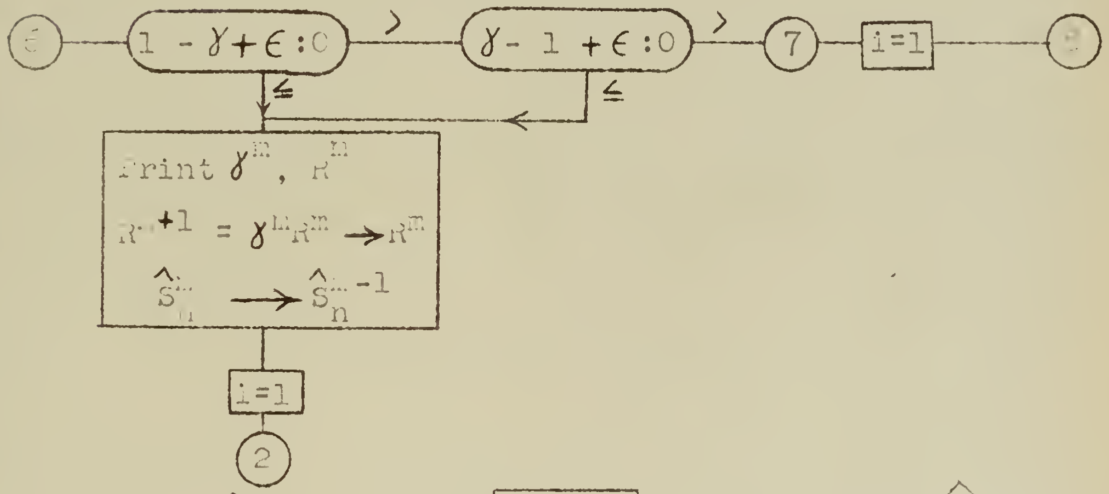
$^{II}S_{n_{in.}} = U_I(x\Sigma_{in.})_I\,{}^IF_n$

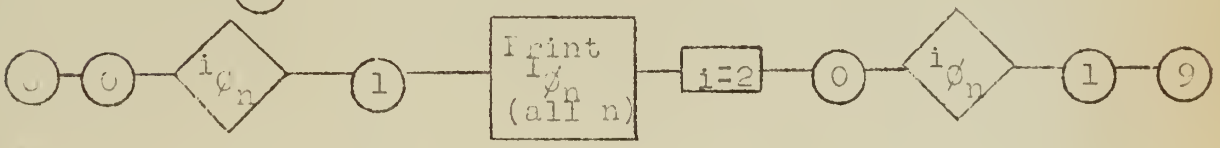
* $^iF_n = \dfrac{^iF_{n+1} + {}^i\beta_n}{^i\alpha_{n+1}}$ starting at $^iF'_{25} = 0$ at reflector outer boundary.

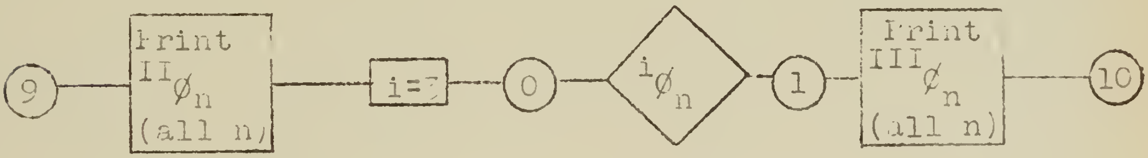
FLOW SHEET FOR FOUR GROUPS

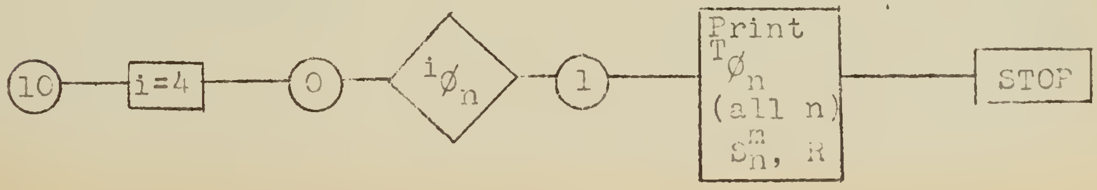
Main Program (Continued)


5
γ = Ŝ^{m-1}_n / S^m_N , Ŝ^m_n = γ S^m_n
1≤n≤25
1 - Ŝ^{m-1}_n / Ŝ^m_n + ε : 0
>
Ŝ^{m-1}_n / Ŝ^m_n - 1 + ε : 0
>
6
≤
≤
Ŝ^m_n → Ŝ^{m-1}_n
i=1
2


6
1 - γ + ε : 0
>
γ - 1 + ε : 0
>
7
i=1
8
≤
≤
Print γ^m, R^m
R^{m+1} = γ^m R^m → R^m
Ŝ^m_n → Ŝ^{m-1}_n
i=1
2

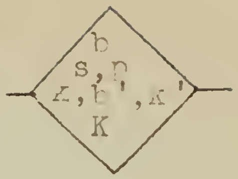

8
0
iφn
1
Print Iφn (all n)
i=2
0
iφn
1
9

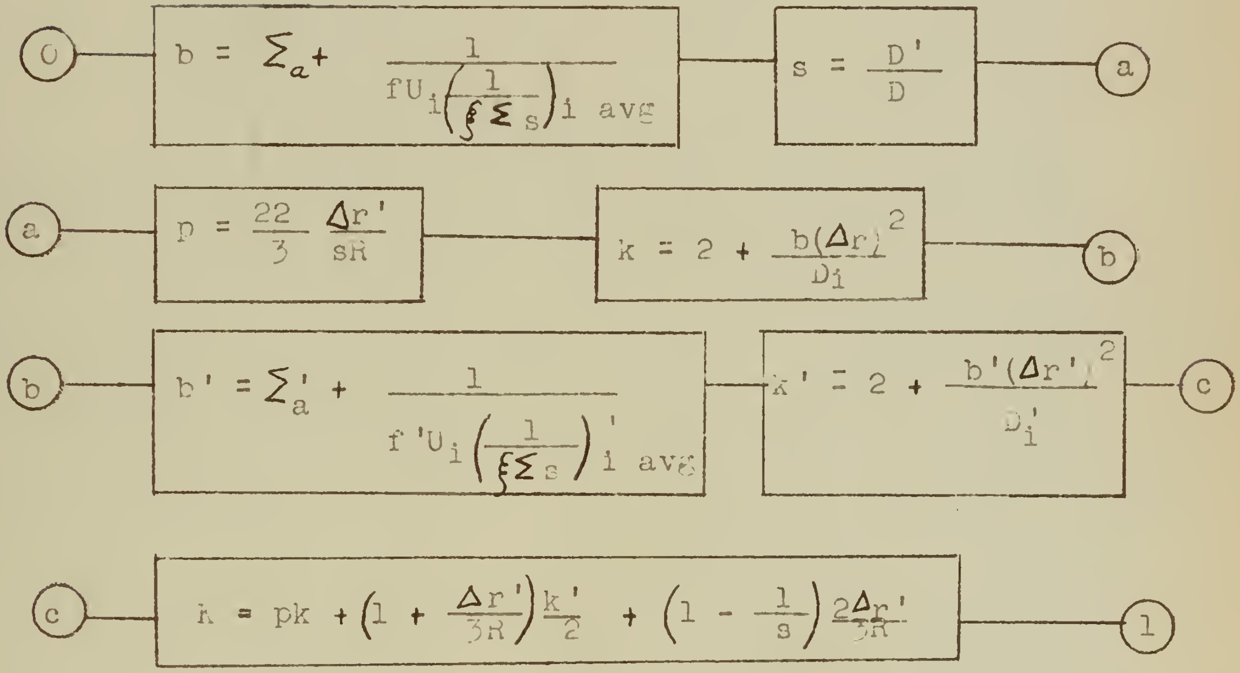

9
Print IIφn (all n)
i=3
0
iφn
1
Print IIIφn (all n)
10


10
i=4
0
iφn
1
Print Tφn (all n) S^m_n, R
STOP

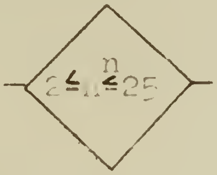
FLOW SHEET FOR FOUR GROUPS

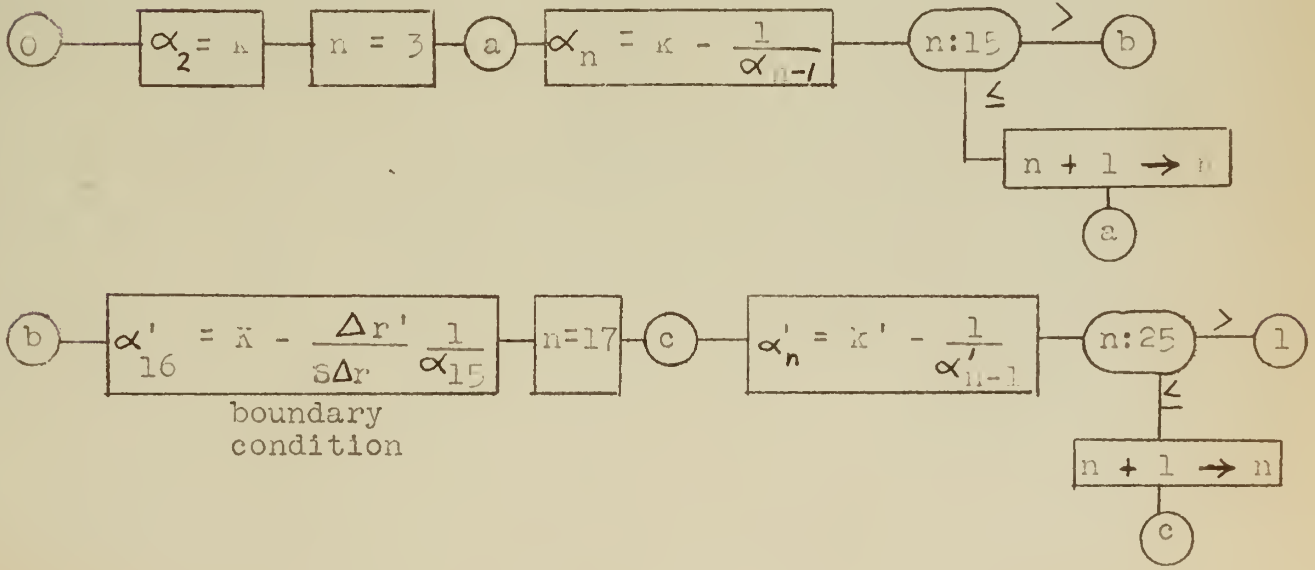
Group Constants Subroutine




0
$b = \Sigma_a + \frac{1}{fU_i\left(\frac{1}{\xi\Sigma_s}\right)_{i\ avg}}$
$s = \frac{D'}{D}$
a
a
$p = \frac{22}{3}\frac{\Delta r'}{sR}$
$k = 2 + \frac{b(\Delta r)^2}{D_i}$
b
b
$b' = \Sigma_a' + \frac{1}{f'U_i\left(\frac{1}{\xi\Sigma_s}\right)'_{i\ avg}}$
$k' = 2 + \frac{b'(\Delta r')^2}{D_i'}$
c
c
$K = pk + \left(1 + \frac{\Delta r'}{3R}\right)\frac{k'}{2} + \left(1 - \frac{1}{s}\right)\frac{2\Delta r'}{3R}$
1

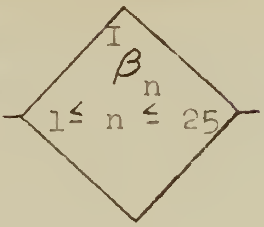
α_n Subroutine

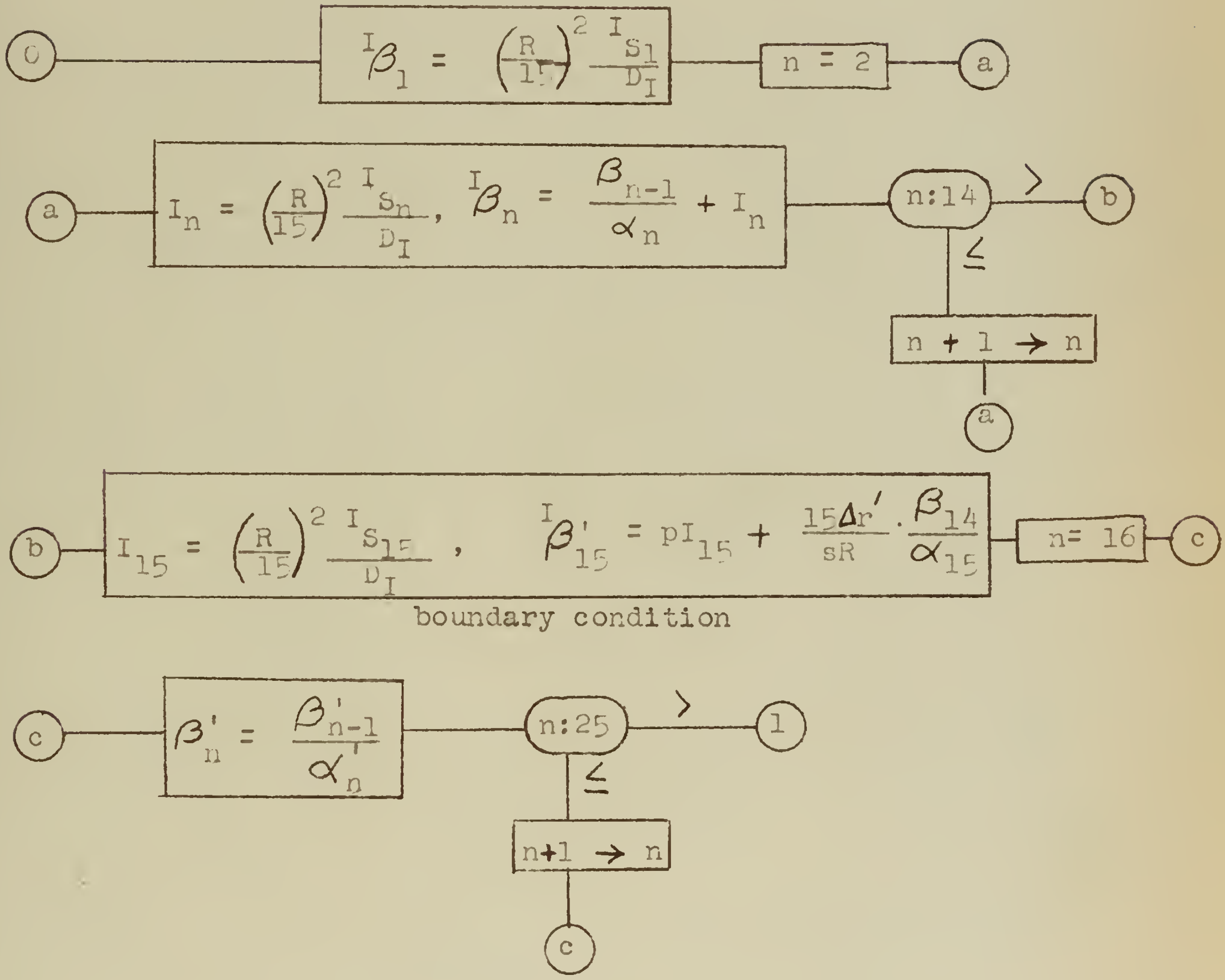

$2 \leq n \leq 25$


0
$\alpha_2 = k$
$n = 3$
a
$\alpha_n = k - \frac{1}{\alpha_{n-1}}$
n:15
>
b
≤
$n + 1 \rightarrow n$
a
b
$\alpha'_{16} = K - \frac{\Delta r'}{s\Delta r}\frac{1}{\alpha_{15}}$
boundary condition
n=17
c
$\alpha'_n = k' - \frac{1}{\alpha'_{n-1}}$
n:25
>
1
≤
$n + 1 \rightarrow n$
c

FLOW SHEET FOR FOUR GROUPS

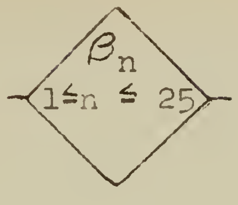
$^I\beta_n$ Subroutine

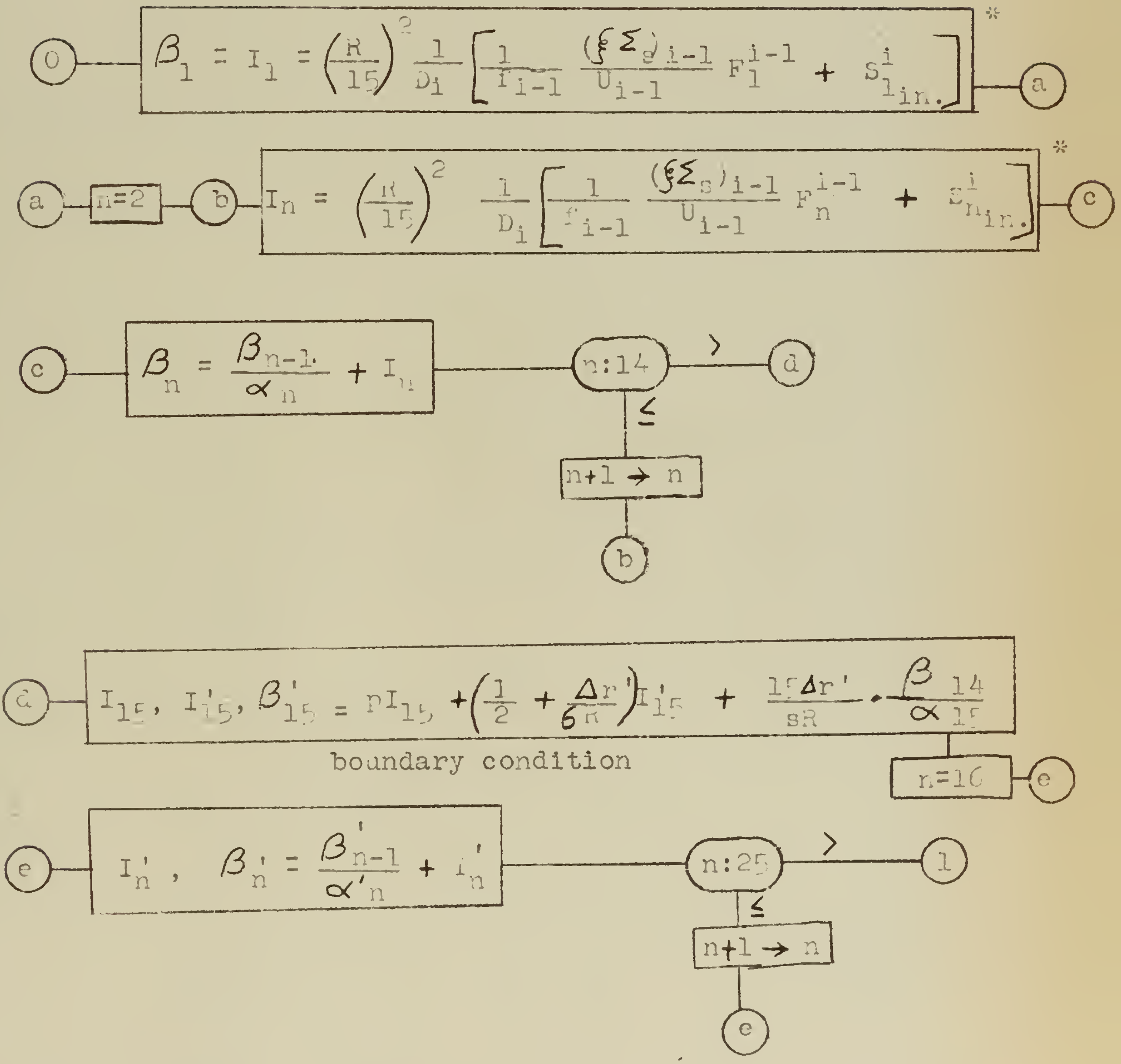

$^I\beta_n$
$1 \le n \le 25$


0
$^I\beta_1 = \left(\frac{R}{15}\right)^2 \frac{I_{S_1}}{D_I}$
$n = 2$
a
a
$I_n = \left(\frac{R}{15}\right)^2 \frac{I_{S_n}}{D_I}, \quad {}^I\beta_n = \frac{\beta_{n-1}}{\alpha_n} + I_n$
n:14
>
b
≤
$n + 1 \to n$
a
b
$I_{15} = \left(\frac{R}{15}\right)^2 \frac{I_{S_{15}}}{D_I}, \quad {}^I\beta'_{15} = pI_{15} + \frac{15\Delta r'}{sR} \cdot \frac{\beta_{14}}{\alpha_{15}}$
boundary condition
$n = 16$
c
c
$\beta'_n = \frac{\beta'_{n-1}}{\alpha'_n}$
n:25
>
1
≤
$n+1 \to n$
c

FLOW SHEET FOR FOUR GROUPS

β_n Subroutine

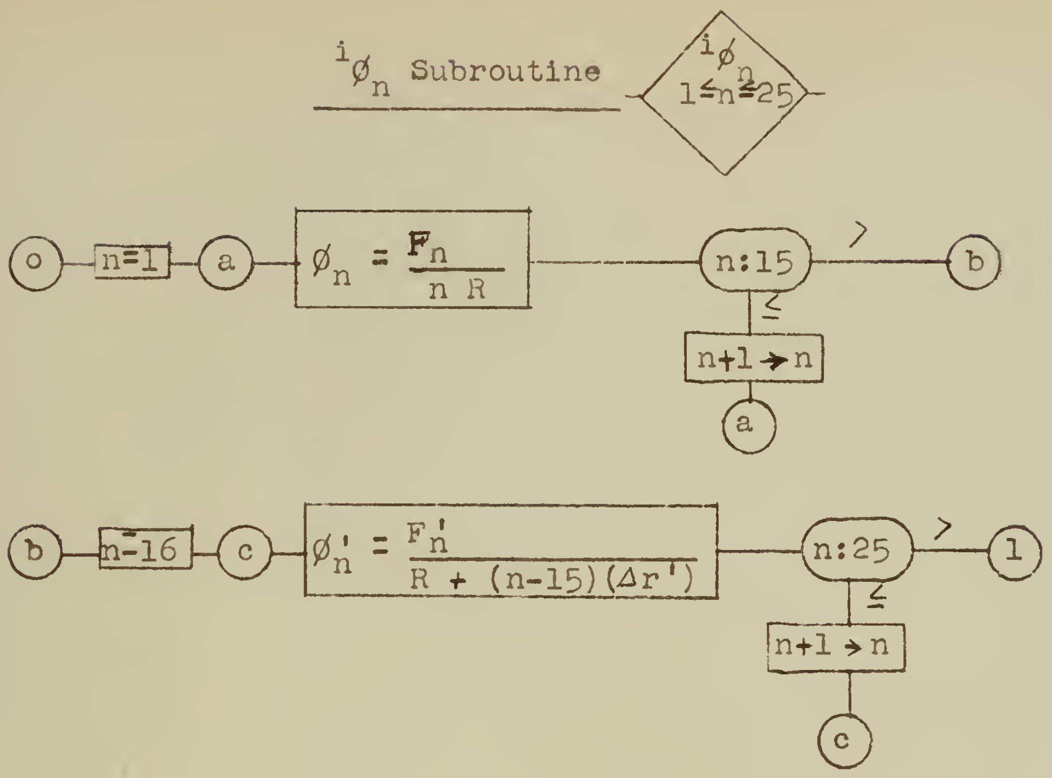

β_n
$1 \leq n \leq 25$


0
$\beta_1 = I_1 = \left(\frac{R}{15}\right)^2 \frac{1}{D_i}\left[\frac{1}{f_{i-1}} \frac{(\xi\Sigma_s)_{i-1}}{U_{i-1}} F_1^{i-1} + S^i_{1_{in.}}\right]$ *
a
a
n=2
b
$I_n = \left(\frac{R}{15}\right)^2 \frac{1}{D_i}\left[\frac{1}{f_{i-1}} \frac{(\xi\Sigma_s)_{i-1}}{U_{i-1}} F_n^{i-1} + S^i_{n_{in.}}\right]$ *
c
c
$\beta_n = \frac{\beta_{n-1}}{\alpha_n} + I_n$
n:14
>
d
≤
n+1 → n
b
d
$I_{15},\ I'_{15},\ \beta'_{15} = pI_{15} + \left(\frac{1}{2} + \frac{\Delta r'}{6R}\right) I'_{15} + \frac{15\Delta r'}{sR} \cdot \frac{\beta_{14}}{\alpha_{15}}$
boundary condition
n=16
e
e
$I'_n,\ \beta'_n = \frac{\beta'_{n-1}}{\alpha'_n} + I'_n$
n:25
>
1
≤
n+1 → n
e

* This form of I_n comes from combination of Equations (1.6), (2.4a), (3.2), and (3.5c) with $\Delta r = R/15$, where R = core radius. (There are 15 core space points.)

FLOW SHEET FOR FOUR GROUPS

$^{i}\phi_n$ Subroutine


$^{i}\phi_n$
$1 \leq n \leq 25$
o
n=1
a
$\phi_n = \frac{F_n}{n\,R}$
n:15
>
≤
b
n+1 → n
a
b
n=16
c
$\phi_n' = \frac{F_n'}{R + (n-15)(\Delta r')}$
n:25
>
≤
1
n+1 → n
c

- - - - - - - - - - -

Memory Assignment (See WWI program following)

Flad*	Contents
p1	Storage of numbers used in numerical operations
p2	Temporary storage
b1	R, $(\nu\Sigma_f)_{II}$, $(\nu\Sigma_f)_{III}$, $(\nu\Sigma_f)_T$, T, $(\Delta r')$ $(\chi\Sigma_{in})_I$, $(\chi\Sigma_{in})_I'$, ϵ, $(\nu\Sigma_f)_I$
i1	Used to indicate number of iterations
c1	Group I constants: D, D', f, f', $(\xi\Sigma_s)_{i\ avg}$, $(\xi\Sigma_s')_{i\ avg}$, Σ_a, Σ_a', $1/v_1$
c2	Group II constants: (Same)
c3	Group III constants: (Same)

---

* A FLAD is a floating address used to locate a particular set of storage registers.

Flad	Contents
c4	Group T constants: (Same)
s1	Assumed source $^{I}S_n^{o}$
s2	Calculated source term from fissions in all groups $\sum_{i=I}^{T}(\nu\Sigma_f F_n)^i = \hat{S}_n^m$
f1	$^{I}F_n \quad 1 \leq n \leq 25$
f2	$^{II}F_n \quad 1 \leq n \leq 25$
f3	$^{III}F_n \quad 1 \leq n \leq 25$
f4	$^{T}F_n \quad 1 \leq n \leq 25$
z1	Operating register for storage of registers ci when group i is being used
z2	Temporary storage for $^{i}\alpha_n$
z3	Temporary storage for $^{i}\beta_n$
z4	Temporary storage for $^{i-1}F_n$
x1	Temporary storage for inelastic scatt. $^{II}(S_n)_{in.}$
r1	Group constants subroutine
r2	α_n subroutine
r3	$^{I}\beta_n$ subroutine
r4	β_n subroutine
r5	$^{I}F_n$ calculation stored in f1
r6	$^{II}F_n$ calculation stored in f2
r7	$^{III}F_n$ calculation stored in f3
r8	$^{T}F_n$ calculation stored in f4
r9, r10	Calculates $(^{II}S_n)_{in.}$ inelastic scattering and stores in x1

Flad	Contents
r12	${}^{1}\phi_n$ subroutine
r13, s10	Iterates for correct spatial distribution of flux
r14, s11	Changes radius and reiterates until criticality is achieved (both r13 and r14 will perform a maximum of 50 iterations before printing out the answer so as to avoid a waste of computer time should the solution be divergent)
x2	Calculates ${}^{I}S_n^{o}$ from the assumed S_n where ${}^{I}S_n^{o} = r_n(S_n)$
a1 and y1	Control for Group I calculations
a2 and y2	Control for Group II calculations
a3 and y3	Control for Group III calculations
a4 and y4	Control for Group T calculations
s8	Calculates $\gamma = \frac{1}{S_N{}^m}$ where N is space point 10
s9	Normalize $\hat{S}_n{}^m = \gamma S_n{}^m$
s10	(See r13)
s11	(See r14)
s12	Puts ${}^{1}\phi_n$ in r5, r6, r7, r8 in place of ${}^{1}F_n$. Prints core power distribution, flux distributions, critical radius, γ, and number of iterations

WHIRLWIND PROGRAM

```
fc TAPE 374-223-008 A HOOVER
(24,6)
t=p2
p1,+1.0    +2.0       +3.0       +4.0       +5.0       +6.0
+7.0       +8.0       +9.0       +10.0      +11.0      +12.0
+13.0      +14.0      +15.0      +16.0      +0.0       +50.0
+50.0      p2,+0.0    +0.0       +0.0       +0.0       +0.0
+0.0       b1,+60.0   +0.001073  +0.01444   +0.052472
+20.0      +2.0       +0.0       +0.00      +0.03      +0.000219
i1,+0.0    +0.0       +0.0       +0.0       c1,+2.209
+1.039     +0.621     +0.68      +0.02158   +0.06884
+0.000106             +0.0       +0.2171    c2,+0.5756
+0.522     +1.528     +1.08      +0.06535   +0.13231   +0.000523
+0.0       +0.16119   c3,+1.184             +0.537     +1.66
+1.05      +0.03813   +0.12968   +0.008951  +0.000522
+0.12484   c4,+1.190             +0.544     +1.0       +1.0
+0.03784   +0.1282    +0.0354    +0.0039144            +0.0
s1,+1.33   +1.31      +1.29      +1.26      +1.22      +1.20
+1.15      +1.10      +1.05      +1.00      +0.95      +0.92
+0.98      +1.14      +1.45      |||||||||
s2,+0.0    DITTO      28r| +0.0
f1,+0.0    DITTO      48r| +0.0  f2,+0.0    DITTO      48r| +0.0
f3,+0.0    DITTO      48r| +0.0  f4,+0.0    DITTO      48r| +0.0
z1,+0.0    DITTO      40r| +0.0  z2,+0.0    DITTO      46r| +0.0
z3,+0.0    DITTO      48r| +0.0  z4,+0.0    DITTO      48r| +0.0
x1,+0.0    DITTO      48r| +0.0  r1,ita54r             icaz1+16
idvz1+4    imrz1+8    iadz1+12   itsz1+18   icaz1+2    idvz1
itsz1+20   icap1+20   imrp1+2    idvp1+4    imrb1+10   idvz1+20
idvb1      itsz1+22   icab1      idvp1+28   imrb1      idvp1+28
imrz1+18   idvz1      iadp1+2    itsz1+24   icaz1+16   idvz1+6
imrz1+10   iadz1+14   itsz1+26   imrb1+10   imrb1+10   idvz1+2
iadp1+2    itsz1+28   icab1+10   idvp1+4    idvb1      iadp1
imrz1+28   idvp1+2    itst       icaz1+22   imrz1+24   its2t
icsp1      idvz1+20   iadp1      imrp1+2    idvp1+4    imrb1+10
idvb1      iad2t      iadt       itsz1+30   isp0       r2,ita25r
icaz1+24   itsz2      isc4       icr13      icsp1      idvz2+c
iadz1+24   itsz2+2+c             ict5r      icsp1      imrp1+28
idvb1      idvz1+20   idvz2+26   imrb1+10   iadz1+30   itsz2+28
isc0       icr9       icsp1      idvz2+28+c            iadz1+28
itsz2+30+c            ict20r     isp0       r3,ita37r
icab1      idvp1+28   imrb1      idvp1+28   idvz1      itst
imrs1
|||||||||| itsz3      isc4       icr13      icat       imrs1+2+c
its2t      icaz3+c    idvz2+c    iad2t      itsz3+2+c
ict11r     icaz1+22   imrt       imrs1+28   its2t      icap1+28
imrb1+10   idvb1      idvz1+20   imrz3+26   idvz2+26   iad2t
itsz3+28   isc5       icr10      icaz3+28+c            idvz2+28+c
itsz3+30+c            ict33r     isp0       r4,ita75r
icab1      idvp1+28   imrb1      idvp1+28   idvz1      itst
icaz1+36   imrz1+40   idvz1+32   its2t      imrz4      iadx1
imrt       itsz3      isc4       icr13      ica2t      imrz4+2+c
```

Whirlwind Program

```
iadx1+2+c  imrt       its4t      icaz3+c    idvz2+c    iad4t
itsz3+2+c             ict17r     ica2t      imrz4+28   iadx1+28
imrt       imrz1+22   its4t      icab1+10   imrb1+10   idvz1+2
itst       icaz1+38   imrz1+40   idvz1+34   its2t      imrz4+28
iadx1+28   imrt       its6t      icab1+10   idvp1+10   idvb1
its8t      icap1      idvp1+2    iad8t      imr6t      its6t
icab1+10   imrp1+28   idvb1      idvz1+20   imrz3+26   idvz2+26
iad4t      iad6t      itsz3+28   isc5       icr10      ica2t
imrz4+30+c            iadx1+30+c            imrt       its8t
icaz3+28+c            idvz2+28+c            iad8t      itsz3+30+c
ict65r     isp0       r5,ita11r             icap1+32   itsf1+48
isc6       icr24      icaf1+48+c            iadz3+46+c
idvz2+46+c            itsf1+46+c            icd2       ict5r
isp0       r6,ita11r             icap1+32   itsf2+48   isc6
|||||||||
icr24      icaf2+48+c            iadz3+46+c            idvz2+46+c
itsf2+46+c            icd2       ict5r      isp0       r7,ita11r
icap1+32   itsf3+48   isc6       icr24      icaf3+48+c
iadz3+46+c            idvz2+46+c            itsf3+46+c
icd2       ict5r      isp0       r8,ita11r             icap1+32
itsf4+48   isc6       icr24      icaf4+48+c            iadz3+46+c
idvz2+46+c            itsf4+46+c            icd2       ict5r
isp0       r9,ita13r             isc3       icr15      icaf1+c
imrb1+12   itsx1+c    ict3r      isc5       icr10      icaf1+30+c
imrb1+14   itsx1+30+c            ict9r      isp0       r10,ita6r
isc2       icr25      icap1+32   itsx1+c    ict3r      isp0
r12,ita21r            icap1+28   idvb1      itst       isc3       icr15
icaz4+c    imrt       idvp1+c    itsz4+c    ict6r      isc5
icr10      icab1+10   imrp1+c    iadb1      its2t      icaz4+30+c
idv2t      itsz4+30+c            ict13r     isp0       r13,isc3
icr15
icas2+c    itss1+c    ict2r      icai1      iadp1      itsi1
icai1+4    iadp1      itsi1+4    isup1+34   icpa1      isps11
r14,isc3
icr15      icas2+c    itss1+c    ict2r      icat       iMOA-123.1234t
isup1      idvp1      iadp1      imrb1      itsb1      iMOA-123.1234c
icai1+2    iadp1      itsi1+2    icai1+6    iadp1      itsi1+6
isup1+36   icpa1      isps12     x2,isc3    icr15      icas1+c
idvp1+18   imrp1+c    itss1+c    ict2r      ||||||||||||
a1,isc0    icr9       icac1+c    itsz1+c    icta1+2    ispr1
ispr2      ispr3      ispr5      ispr9      y1,isc3    icr15
icab1+18   imrf1+c    itss2+c    icty1+2    a2,isc0    icr9
icac2+c    itsz1+c    icta2+2    isc1       icr4       icac1+4+c
itsz1+32+c            icta2+7    icac1+16   itsz1+40   isc2
icr25      icaf1+c    itsz4+c    icta2+14   ispr1      ispr2
ispr4      ispr6      ispr10     y2,isc3    icr15      icab1+2
imrf2+c    iads2+c    itss2+c    icty2+2    a3,isc0    icr9
icac3+c    itsz1+c    icta3+2    isc1       icr4       icac2+4+c
itsz1+32+c            icta3+7    icac2+16   itsz1+40   isc2
icr25      icaf2+c    itsz4+c    icta3+14   ispr1      ispr2
ispr4      ispr7      y3,isc3    icr15      icab1+4    imrf3+c
```

```
iads2+c    itss2+c    icty3+2    a4,isc0    icr9       icac4+c
itsz1+c    icta4+2    isc1       icr4       icac3+4+c
itsz1+32+c            icta4+7    icac3+16   itsz1+40   isc2
icr25      icaf3+c    itsz4+c    icta4+14   ispr1      ispr2
ispr4      ispr8      y4,isc3    icr15      icab1+6    imrf4+c
iads2+c    itss2+c    icty4+2    s8,icap1   idvs2+18   itst
s9,isc3    icr15      icat       imrs2+c    itss2+c
icts9+2    s10,isc3   icr15      icas1+c    idvs2+c    its2t
icap1      iadb1+16   isu2t      icpr13     icsp1      iadb1+16
iad2t      icpr13     icts10+2   s11,icab1+16          isut
iadp1      icpr14     icat       isup1      iadb1+16   icpr14
iFOR       +5         +0         +25        s12,isc2   icr25
icaf1+c    itsz4+c    ict2r      ispr12     isc2       icaz4+c
itsf1+c    ict7r      isc2       icaf2+c    itsz4+c    ict11r
ispr12     isc2       icr25      icaz4+c    itsf2+c    ict17r
isc2       icaf3+c    itsz4+c    ict21r     ispr12     isc2
icaz4+c    itsf3+c    ict26r     isc2       icaf4+c    itsz4+c
ict30r     ispr12     isc2       icaz4+c    itsf4+c    ict35r
isc2       icr25      icaf1+c    idvf4      itsf1+c
iMOA-p123.12345f
ict40r     isc2       icaf2+c    idvf4      itsf2+c    iMOA
ict46r     isc2       icaf3+c    idvf4      itsf3+c    iMOA
ict52r     icaf4
its4t      isc2       icaf4+c    idv4t      itsf4+c    iMOA
ict60r     icai1      iMOA       icai1+2    iMOA       icab1
iMOA       isc3       icr15      icas2+c    imrp1+18   idvp1+c
iMOA       ict73r
iSTOP
i START AT x2
```

Table IV of Appendix B gives the Whirlwind input data for runs (1) through (10). The Whirlwind program for run (8) has been given on the preceeding pages. The storage assignments for the input data are as follows (using run (8) as an example):

FLAD	Numerical Value	Description of quantity	
bl,	60.0	R, assumed core radius (cm)	
	0.001073	$(\nu\Sigma_f)_{II}$	
	0.01444	$(\nu\Sigma_f)_{III}$	
	0.052472	$(\nu\Sigma_f)_T$	
	20.0	T, reflector thickness (cm)	
	2.0	$\Delta r' = T/10$ (cm)	
	6.0	$(\chi\Sigma_{in.})_I$, inelastic scattering (core)	
	0.0	$(\chi\Sigma_{in.})_I$, inelastic scattering (reflector)	
	0.03	ϵ , iteration criteria for 3% accuracy	
	0.000219	$(\nu\Sigma_f)_I$	
cl,	2.209	D, diffusion coefficient, core,	Group I
	1.039	D', diff. coef., reflector	"
	o.621	f, f factor, core	"
	0.68	f', f factor, reflector	"
	0.02158	$(\xi\Sigma_s)$ for core	"
	0.06084	$(\xi\Sigma_s)'$ for reflector	"
	0.000106	Σ_a , for core absorption	"
	0.0	Σ_a' , absorption for reflector	"
	0.2171	$1/U_I$, reciprocal of Group I lethargy width	
c2,	(See program)	Same quantities as in cl,	Group II
c3,	(See Program)	" " " " " ,	Group III
c4,	(See program)	" " " " " ,	Group T (thermal)

REFERENCES

(1) "Comprehensive System Manual," Digital Computer Lab., Massachusetts Institute of Technology, Cambridge, Massachusetts, December, 1955.

(2) S. Glasstone and M. C. Edlund, "The Elements of Nuclear Reactor Theory," D. Van Nostrand Co., Inc., New York, 1954 (188).

(3) Glasstone and Edlund, Op. Cit., (229).

(4) R. Ehrlich and H. Hurwitz, Jr., "Nucleonics," 12 (2):23, 1954.

(5) S. Glasstone, "Principles of Nuclear Reactor Engineering," D. Van Nostrand Co., Inc., New York, 1955 (111, Fig. 2.22).

(6) Glasstone and Edlund, Op. Cit., (30).

(7) "Reactor Handbook," Volume 1, Physics, AECD-3645 (146, Fig. 1.2.33).

CHAPTER II

Numerical Integration Method

R. W. Kennedy

I. Derivation of Equations*

The following diffusion equation is assumed to apply:

$$\frac{1}{3\Sigma_{tr_i}}\nabla^2 \phi_i - \Sigma_{abs_i}\phi_i = -\frac{\partial \xi \Sigma_{s_i}\phi_i}{\partial \log_e E} \qquad (1)$$

The subscript i refers to a particular energy group. In this case (considering six energy groups) i ranges from 1 to 6, where group 6 neutrons are of highest energy, group 1 of lowest (thermal).

For group 6 Equation (1) is modified to include a fission neutron source. All fission neutrons are assumed to be born in group 6.

Equation (1) is to be made dimensionless, converted to a standard form (to allow the computer to use one standard calculation form for all groups) and integrated. It is first multiplied by (Σref/Σref), where Σref is a reference macroscopic cross-section, here taken as the thermal transport cross-section of the reflector. Equation (1) becomes:

---

*This derivation is based on a method developed by A. S. Thompson (J. Appl. Phys. 22, 1223) with modifications introduced to include leakage of the fast group (6) and in order to exploit computer advantages. The original method was developed for hand calculations.

$$\frac{1}{3\,\Sigma_{ref}}\;\frac{\Sigma_{ref}}{\Sigma_{tr_i}}\nabla^2\phi_i - \frac{\Sigma_{abs}\,\Sigma_{ref}}{\Sigma_{ref}}\phi_i = -\frac{\partial\xi\Sigma_{ref}\frac{\Sigma_{s_i}}{\Sigma_{ref}}\phi_i}{\partial\log_e E} \quad (2)$$

For convenience the following dimensionless cross-sections are defined:

$a_i = \Sigma_{ref}/\Sigma_{tr_i}$

$b_i = \Sigma_{abs_i}/\Sigma_{ref}$

$d_i = \Sigma_{s_i}/\Sigma_{ref}$

$e_i = \Sigma_{f_i}/\Sigma_{ref}$

With these definitions Equation (2) becomes:

$$\frac{a_i}{3\,\Sigma_{ref}}\nabla^2\phi_i - b_i\,\Sigma_{ref}\,\phi_i = -\frac{\partial\xi\, d_i\,\Sigma_{ref}\,\phi_i}{\partial\log_e E} \qquad (3)$$

Next, to simplify the energy variable the following definitions are established:

$$\beta = \frac{\log_e(E_{fiss}/E_i)}{\log_e(E_{fiss}/E_{th})}$$

$$c = (\log_e E_{fiss}/E_{th})^{-1}$$

Thus,

$$\begin{aligned}\partial/\partial\log E &= \partial/\partial\beta \times \partial\beta/\partial\log E\\ &= \partial/\partial\beta\left\{-1/\log\frac{E_{fiss}}{E_{th}}\right\}\\ &= -c\,\partial/\partial\beta\end{aligned}$$

and the operator $(-c\,\partial/\partial\beta)$ may be substituted for the operator $(\partial/\partial\log_e E)$ in Equation (3), giving:

$$\frac{a_1}{3\Sigma_{ref}}\nabla^2\phi_1 - b_1\Sigma_{ref}\phi_1 = \frac{\partial \xi d_1 c \Sigma_{ref}\phi_1}{\partial\beta} \qquad (4)$$

Finally, the operator ∇^2, which is given in spherical coordinates (with radial variations only) by

$$\nabla^2 = \partial^2/\partial r^2 + \frac{2}{r}\partial/\partial r$$

becomes, upon considering a dimensionless space parameter r/R (where R is the total radius, including core, reflector, and extrapolation distance):

$$\nabla^2 = \frac{\partial^2}{\partial(r/R)^2 R^2} + \frac{2}{(r/R)R}\,\frac{\partial}{\partial(r/R)R}$$

or

$$R^2\nabla^2 = \frac{\partial^2}{\partial(r/R)^2} + \frac{2}{(r/R)}\,\frac{\partial}{\partial(r/R)}$$

Thus Equation (4) becomes

$$\frac{a_1}{3\Sigma_{ref}R^2}(R^2\nabla^2)\phi_1 - b_1\Sigma_{ref}\phi_1 = \frac{\partial \xi c d_1 \Sigma_{ref}\phi_1}{\partial\beta} \qquad (5)$$

or, since the operator $(R^2\nabla^2)$ in r-space is equivalent to the operator (∇^2) in r/R-space:

$$\frac{a_1}{3\Sigma_{ref}R^2}\nabla^2\phi_1 - b_1\Sigma_{ref}\phi_1 = \frac{\partial \xi c d_1 \Sigma_{ref}\phi_1}{\partial\beta} \qquad (6)$$

Upon multiplying Equation (6) through by $3(\Sigma_{ref}R^2)$, and defining

$$\lambda = 3(\Sigma_{ref}R)^2$$

Equation (6) becomes:

$$a_1\nabla^2\phi_1 - b_1\lambda\phi_1 = \frac{\partial \xi\lambda c d_1\phi_1}{\partial\beta} \qquad (7)$$

Note that the parameter λ depends on the total radius of the reactor and is thus not originally known. Instead it is assumed (based on one-group calculations) and later corrected

if the assumed radius is sub- or super-critical.

The derivative on the right hand side of Equation (7) may be approximated:

$$\frac{\partial \lambda \xi c d_i \varphi_i}{\partial \beta} \approx \frac{\lambda \xi c d_i \varphi_i - \lambda \xi c d_{i+1} \varphi_{i+1}}{\Delta \beta}$$

so that: $$a_i \nabla^2 \phi_i - b_i \lambda \phi_i - \frac{\lambda \xi c d_i \varphi_i}{\Delta \beta} = - \frac{\lambda \xi c d_{i+1} \varphi_{i+1}}{\Delta \beta} \quad (8)$$

The left hand side of Equation (8) represents loss from the i'th energy group by leakage, absorption, and slowing down. The right hand side represents the input to the i'th group consisting of neutrons slowed from the next higher group.

For i=1 (thermal group) Equation (8) is modified to: $$a_1 \nabla^2 \phi_1 - b_1 \lambda \phi_1 = \lambda \xi c d_2 \phi_2 \quad (8a)$$ since no neutrons are lost from the thermal group by slowing down.

The input term in Equation (8) is modified for energy group 6 (fast) to include fission:

$$a_6 \nabla^2 \phi_6 - b_6 \lambda \phi_6 - \frac{\lambda \xi c d_6 \phi_6}{\Delta \beta} = \frac{\lambda \nu e_1 c \phi_1}{\Delta \beta} + \sum_{i=2}^{6} \lambda \nu e_i \phi_i \quad (9)$$

(The fission of thermal neutrons must be weighted by an energy factor since all other fluxes in (9) are per unit energy interval and must be multiplied by this energy interval to become the total flux. The equation is simplified by dividing through by the energy interval with the result that it appears as a denominator in the thermal term.)

Equations (8), (8a), and (9) are of the form

$$(a_i \nabla^2 - g^2) \phi_i = h$$

where g^2 combines all loss terms except leakage, and h represents an input term. For example, considering energy group 1, g^2 is given by $(b_1\lambda)$ and h by $(-\lambda\xi d_2\phi_2)$ from (8a).

If this form of Equations (8), (8a), and (9) is multiplied through by $a_i(r/R)^4$ and the operator $a_i\nabla^2$ replaced by its equivalent $(a_i \frac{1}{(r/R)^2}\, d/d(r/R)\, (r/R)^2\, d/d(r/R))$ the following expression (with $x = r/R$) results:

$$a_i x^4 \left\{ (1/x^2) d/dx (a_i x^2) d/dx - g^2 \right\} \phi_i = a_i x^4 h$$

or, with $D_i = a_i x^2\, d/dx$,

$$(D_i^{\,2} - a_i x^4 g^2)\, \phi_i = a_i x^4 h \qquad (10)$$

Equation (10) is of the type

$$(D^2 - a x^4 g^2)\, Y = Z$$

which may be factored into

$$(D + f)(D - f)\, Y = Z \qquad (11)$$

provided f satisfies

$$Df + f^2 - a x^4 g^2 = 0 \qquad (12)$$

This arises from

$$(D + f)(D - f)\, Y = Z = (D^2 - a x^4 g^2)\, Y$$

$$D^2Y + fDY - D(fY) - f^2Y = D^2Y - a x^4 g^2 Y$$

$$D^2Y + fDY - (Df)Y - f(DY) - f^2Y = D^2Y - a x^4 g^2 Y$$

or, $$(Df)Y + (f^2)Y - (a x^4 g^2)Y = 0$$

which gives Equation (12) specifying f.

In Equation (11) let $(D - f)Y = U$, so that

$$(D + f)U = Z \qquad (13)$$

where $$(D - f)Y = U \tag{14}$$

Integration of Equation (14) gives

$$Y = \exp\left(\int_0^x f\, dx'/ax'^2\right)\left\{\int_0^x \exp\left(-\int_0^{x'''} f\, dx''/ax''^2\right)U\, \frac{dx'''}{ax'''^2} + C_1\right\} \tag{15}$$

and from Equation (13)

$$U = \exp\left(-\int_0^x \frac{f\, dx'}{ax'^2}\right)\left\{\int_0^x \exp\left(\int_0^{x'''} \frac{f\, dx''}{ax''^2}\right) Z\, \frac{dx'''}{ax'''^2} + C_2\right\} \tag{16}$$

which combine to give (writing $m = \int_0^x f\, dx'/ax'^2$)

$$Y = m \int_0^x (1/m^2)\left[\int_0^{x'''} m\, Z\, dx''/ax''^2 + C_2\right] dx'''/ax'''^2 + C_1 \tag{17}$$

The constants C_1 and C_2 are determined by the conditions that

$Y' = 0$ at $x = 0$ and

$Y = 0$ at $x = 1$

which is equivalent to requiring neutron flux symmetry at the reactor center, and requiring the flux to vanish at the extrapolated radius.

By the first condition C_2 must be zero. By the second C_1 is given by

$$C_1 = - \int_0^1 (1/m^2)\left[\int_0^{x''} m\, Z\, dx'/ax'^2\right] dx''/ax''^2$$

so that Equation (17) becomes

$$Y = m\left\{\int_1^x \left[(1/m^2) \int_0^{x''} m\, Z\, dx'/ax'^2\right] dx''/ax''^2\right\} \tag{18}$$

Solution of Equation (18) requires prior calculation of f from Equation (12) with the initial condition that f(0) = 0 from (11) and (10). Equation (12) is written

in terms of differences:

$$ax^2 d/dx(f) + (f)^2 - ax^4 g^2 = 0$$

or, with subscript 1 indicating space point x and subscript 2 indicating space point $(x + \Delta x)$:

$$(f_2 - f_1)/\Delta x = \tfrac{1}{2}(g_2^2 x_2^2 + g_1^2 x_1^2) - \tfrac{1}{2}(f_2^2/a_2 x_2^2 + f_1^2/a_1 x_1^2)$$

Solving by quadratic formula gives

$$f_2 = -a_2 x_2^2/\Delta x + a_2^{\frac{1}{2}} x_2 \Big\{ a_2 x_2^2/(\Delta x)^2 + 2f_1/\Delta x + g_2 x_2^2 + + g_1 x_1^2 - f_1^2/a_1 x_1^2 \Big\}^{\frac{1}{2}} \qquad (19)$$

Factors in Equations (18) and (10) are summarized for clarity:

Energy Group	h (input)	g^2(loss)
1 (th)	$(-\lambda \xi cd_2)\phi_2$	$b_1\lambda$
2	$(-\frac{\lambda \xi cd_3}{\Delta\beta})\phi_3$	$b_2\lambda + (cd_2/\Delta\beta)\lambda\xi$
3	$(-\frac{\lambda \xi cd_4}{\Delta\beta})\phi_4$	$b_3\lambda + (cd_3/\Delta\beta)\lambda\xi$
4	$(-\frac{\lambda \xi cd_5}{\Delta\beta})\phi_5$	$b_4\lambda + (cd_4/\Delta\beta)\lambda\xi$
5	$(-\frac{\lambda \xi cd_6}{\Delta\beta})\phi_6$	$b_5\lambda + (cd_5/\Delta\beta)\lambda\xi$
6	$-\frac{\nu e_1 c\phi_1}{\Delta\beta} - \sum_{i=1}^{6} \nu e_i \phi_i$	$b_6\lambda + (cd_6/\Delta\beta)\lambda\xi$

II. Programming of Problem for Computer

Whirlwind I (WWI) is a digital computer capable of performing simple mathematical operations (addition, sub-

traction, multiplication, division) with extreme rapidity and under the control of a set of instructions, called the program, which may be stored in the computer. WWI normally operates with numbers that are all less than unity, but, by accepting a loss in computer speed and storage capacity, may be operated with unscaled numbers (thereby decreasing enormously the difficulty of setting up the program) in the Comprehensive System (CS system), as is done in this problem.

In the CS mode there are 1360 registers available for storage of either instructions or numbers. An instruction occupies one register, but a number occupies two. Of the two registers containing a number, the first carries a part of the significant figures which represent the number expressed in exponential form, while the second carries the exponent and remaining significant figures. For example, the number 123.4 may be expressed as $.1234 \times 10^3$, with .123 stored in the first register, and the exponent 3 and the remaining significant .0004 stored in the second. Actually, WWI operates with binary numbers, but an understanding of this feature is not essential to the understanding of the problem since the computer accepts decimal numbers from the programmer and returns decimal number answers.

The 1360 registers which comprise the internal storage capacity of WWI in the CS mode are called the magnetic core memory (MCM). External storage is available

in the form of magnetic drums and tape but acquisition time is large in comparison with the time required for the computer to obtain a number stored in MCM and is therefore not used in this problem.

Any problem then is limited to a set of numbers and instructions which occupy not more than 1360 registers. In this problem these registers are used (starting at the first register available, which is number 32) as follows:

Registers 32-209: working storage (for numbers which are calculated during the problem and must be temporarily stored)

Registers 210-469: storage for 20 values of the neutron flux in each of the six energy groups

Registers 470-509: storage for 20 space points (r/R = .05, .10, .15 etc.)

Registers 510-597: cross-section storage

Registers 598-639: storage of constants other than cross-sections

Registers 640-end: storage for the program instructions

Actually the program for this problem uses only some 1250 registers so that 110 are available for later expansion of the problem if desired.

For the purposes of this problem there are sixteen

commands which WWI obeys in the CS mode. They each affect an operation, designated by three letters, and an address. The address may consist simply of the number of a register which contains, for example, the number which the computer is to obtain from MCM. However, since a programmer cannot know in advance exactly how many registers to set aside, it is frequently difficult to specify a register number as an address. To eliminate this problem, WWI is designed to allow use of indefinite addresses, called flads (for floating addresses) and represented by a letter-number combination such as a1, a2, b1, b2, etc. Thus, if a number is to be stored at the end of the program it is tagged with a flad and the flad used in place of the actual register number. WWI simply counts the registers as the program is read in and remembers the actual address of the flad. Then, whenever this flad is referred to later WWI substitutes the proper address.

The three-letter portion of the WWI commands generally calls for a certain arithmetic operation. All of these occur in the Multiple Register Accumulator (MRA). For example, if a number in storage is to be multiplied by another stored number, the first is taken from MCM to the MRA. The second is then obtained, the required product formed in the MRA and then transferred to storage. Various operations, as explained below, may either preserve or destroy the originally stored quantities.

The sixteen instructions used in this problem are the following:

Instruction	Meaning	Operation
its al	transfer the contents of MRA to register al	the number in the MRA is transferred to register al; the former contents of al are lost; the contents of MRA are unchanged
iex al	exchange the contents of MRA and register al	the contents of the MRA become the new contents of register al, and vice versa
ica al	clear MRA and add to it the contents of register al	the MRA is cleared (and its contents lost) and its new content is the number stored in al; al remains unchanged
ics al	clear MRA and subtract from it the contents of al	as above except that MRA acquires the negative of al; al is unchanged
iad al	add the contents of al to MRA	the contents of al, plus the original contents of MRA are added and the result stored in MRA; al is unchanged
isu al	subtract the contents of al from MRA	the contents of al are subtracted from MRA and the result stored in MRA; al is unchanged
imr al	multiply MRA by al and round off	the number in MRA is multiplied by the number in al; the result rounded off to the maximum number of significant figures which can be stored (about 8 decimally) is retained in MRA; al is unchanged
idv al	divide MRA by contents of al	MRA is divided by contents of al, with the result stored in MRA; al is unchanged

The following two instructions provide transfer of control to the instruction stored in the address of the instruction. WWI normally operates sequentially from one register to the next. These two instructions circumvent this sequential operation. As an example, the last instruction in an iterative problem might contain a transfer of control back to the first in order to perform the iteration.

Instruction	Meaning	Operation
isp al	control transfer	take the next instruction from register al and proceed from there
icp al	conditional control transfer	as above, if the contents of MRA are negative; otherwise, ignore, and take the next instruction in sequence

The next five instructions have to do with operations performed with a counter. In WWI a counter may be set aside to allow repetition of an instruction, or set of instructions, a specified number of times. The specified number is the criterion, the number of times the operation has been performed is the index. WWI sets aside both criterion and index (with index initially zero), cycles the index once each time the operation is repeated until, upon comparison the index is found to equal the criterion (or if the index exceeds the criterion, as pointed out below) at which time the computer proceeds to the next instruction in sequence.

Instruction	Meaning	Operation
isc j	select counter j	select counter j for following operations with counter

(selecting a counter j, by isc j, does not in itself set criterion and index; this must be done by an additional step prior to the first use of counter j.)

Instruction	Meaning	Operation
icr m	cycle reset	set criterion equal to m, index zero
ict al	cycle count	increase the index register by one and compare with criterion; if the index is less return to register al; otherwise take the next instruction in sequence
ici n	cycle increase	increase the index by n (this may set index greater than criterion, but is useful in operating with more than one counter)
icd n	cycle decrease	decrease index by n

(Note that the counting instruction, ict al, is the only counter command that transfers control. It is therefore usually the last instruction in an operation which is to be repeated.)

During operation with a counter WWI may treat numbers (or, in special cases, instructions) which are stored in sequence if +c is appended to the address of the first number to be treated. For example, to place ten numbers which are stored consecutively in ten other consecutive registers it suffices to select a counter with criterion ten,

index zero, obtain and transfer the first number, with +c appended to the address section of the obtain and transfer instructions, and transfer control back to the first instruction (in, say, register b_1) with an ict bl. Each time the computer notes the +c in the address it sets c equal to twice the index of the counter in use and thus transfers the ten desired numbers. The computer sets c to twice the index because of the fact that the numbers occupy two registers. This however is not the case if a +c is added to the address section of an isp command. Here the address must be another instruction, occupying one register, so the computer conveniently sets c equal to the index.

The final instruction is used in connection with subroutines within the program. Identical operations which are to be performed at different points in the program may be set up as a subroutine in order to avoid duplication of instruction storage each time the required calculation is used. Then, at each point the subroutine is needed, an isp instruction transfers control to the subroutine. However, to leave the subroutine and return to the proper point in the main program an isp instruction at the end of the subroutine must have the address of the register immediately following the register which originally transferred control to the subroutine. This is accomplished by the final instruction to be considered here.

Instruction	Meaning	Operation
ita a1	transfer address	the address section of the instruction in a1 is changed to the register immediately after the register containing the latest isp instruction which has been obeyed

For example, to obtain the square root of a number at several points in the main program, a square root subroutine may be set up at the end of the program. Then, each time the operation is to be performed the number is placed in the MRA, control transferred to the subroutine, the root calculated in the subroutine and placed in the MRA, and control transferred back to the main program to the register immediately following the original transfer point.

In the subroutine the first instruction would be ita b1, with b1 the last register in the subroutine. In b1 would be stored isp ---. Thus the ita instruction would cause transfer of control back to the main program at the proper point.

III. Programming of Specific Problem.

The problem to be solved here, derived in Part I of this chapter, may be stated as

$$Y(x) = \exp\left(\int_0^x \frac{f\ dx'}{ax'^2}\right)\left[\int_1^x \exp\left(-2\int_0^{x^{iv}} \frac{f\ dx'}{ax'^2}\right)\left\{\int_0^{x^{iv}} \exp\left(\int_0^{x'''} \frac{f\ dx''}{ax''^2}\right) \frac{Z dx'''}{ax'''^2}\right\} \frac{dx^{iv}}{ax^{iv\,2}}\right]$$

where x is a dimensionless space parameter r/R total

a is a stored dimensionless cross-section;

Z is the source term, representing fission neutrons (for the highest energy group) or slowed neutrons (for all lower energy groups); and

f is calculated from a finite difference approximation.

Fundamentally the computer proceeds by the following steps:

(1) Calculate $f(x)$;

(2) Divide $f(x)$ by ax^2 and integrate from 0 to x by trapezoidal summation;

(3) Calculate and store $\exp\left[\int_0^x f\, dx/ax^2\right]$;

(4) Calculate source term Z, initially from an assumed flux and critical radius;

(5) Divide Z by ax^2 and multiply by the exponential computed in step (3), preserving the stored exponential for later re-use;

(6) Integrate $\int_0^x \left[(Z/ax^2)(\exp \int_0^x f\, dx/ax^2)\right] dx$;

(7) Divide the above integral by the square of the exponential (still preserving the exponential) and by ax^2;

(8) Reverse the order of storage of the quantity determined in step (7) so that the integral from 1 to x may be determined (this step is necessary since MI counters increase

rather than decrease following each repetition).

Step (8) effectively cancels the minus sign which appears in the input term to each group, with the result that the calculated neutron flux is positive, as it should be.

(9) Reverse the order of storage of the integrals determined in step (8) and multiply by the exponential term.

(10) Store the results of step (9) and proceed to the next energy group calculations, changing f, a, and Z in steps (1) through (9).

Following the sixth application of these steps the new thermal flux has been computed; WWI proceeds then to the next step:

(11) Compute a new critical radius parameter λ':

$$\lambda' = \left[\int_0^1 \phi_{th}(x)dx \Big/ \int_0^1 \phi'_{th}(x)dx\right]\lambda$$

where the primed quantities are the newly computed values.

Step (11) is based on the fact that once the proper solution for the flux is determined ϕ_{th} will not change with more iterations, as long as the radius is equal to R_{crit}. On the other hand, if the radius assumed was sub- or super-critical ϕ'_{th} will be less than or more than ϕ_{th} (assumed) respectively.

(12) Compare the two values of the radius parameter; if they differ by more than an acceptable error recalculate, based on the new values of flux and radius; if the reactor is critical, as evidenced by a specified small variation in λ

print out specified information (e.g., flux, radius, power distribution) and stop.

The computer may be stopped at the end of a problem by an iSTOP instruction. Alternatively, if more problems are to be considered they may be listed after the first problem and the computer stopped with the special instruction sp26. When it reaches this command at the end of the first problem the computer automatically reads in the remaining problems consecutively. In this way a considerable saving in time may be realized, since manual effort (otherwise required to start each problem) is minimized and since many of the constants which are already in storage will not change from problem to problem, thus decreasing the time required for the computer to convert the input decimal numbers into binary numbers. After the final problem an iSTOP command may be read in to stop the computer. Otherwise the operator will stop the computer manually when, upon completion of the last problem, the computer, under the sp26 instruction, calls for more information.

As mentioned in part II of this chapter, the numerical values of this problem are stored in the first portion of MCM. The first instruction is stored in register 640. Development of the program for this problem will start at this point with numbers specified as they appear.

Instruction	Operation
a1,isc0 icrpp1 isc1 icrpp2 isc2 icr20 isc3 icr20 isc4 icr6	Sets counters: 0 for iteration over core region, 1 over reflector, 2 and 3 over entire reactor (20 space points) and 4 over six energy groups; pp1 and pp2 are preset parameters which are set at the opening of the program in order that the rest of the program may remain unchanged
ica636 its598	Stores assumed radius parameter in 598; 636 contains latest calculated λ
isc2 a3,ica430+c imr638 its430+c icta3	Scales ϕ_{th} to any desired center-line value (value initially set in register 638)
f2,isc0 a2,ica510 imr254+c its34+c ica512 imr298+c iad34+c its34+c ica514 imr342+c iad34+c its34+c ica516 imr386+c iad34+c its34+c ica518 imr430+c iad34+c imr598 imr470+c imr470+c its34+c icta2	Selects counter 0 to iterate over core; double-length registers 510 through 518 contain fission parameters; 254 through 468 contain fluxes; input term Z to fast energy group is determined as the sum of each ($\nu_2 \Sigma_f \phi_i$) multiplied by (λ) in 598, x^2 (x in 470 through 508)

ica574 its618 ica576 its620 ica530 its622 ica532 its624	Stores loss parameters $(b+\xi\, cd/\Delta\beta)$ and a $(\Sigma ref/\Sigma tr)$ of group 6 in working registers 618-624 (this is done in order that other group parameters may later be placed in these same working registers and the remainder of the program used for all energy groups)
ispm1	causes a jump around the next 3 instructions which provide entry for groups 1-5
a4,isc4 ictm1	a4 is flad to label entry point for groups 1-5; next register jumped unless new ϕ_{th} has been calculated
ispx6	jumps to ϕ_{th} subroutine in x6 when new ϕ_{th} has been computed
m1,isc0 a5,ica618 imr470+c imr470+c its78+c icta5	calculates ax^2 for core region
isc1 a6,ica620 imr470+pp3+c imr470+pp3+c its78+pp3+c icta6	as above, for reflector region; pp3 is set at the opening of the program to twice (because of double-length registers) the estimated value of $Rcore/R_{tot}$; thus, in order to specify a reflector core combination it is only necessary to change pp1,2,3, not program
ica620 iad618 imr614 imr470+pp3 imr470+pp3 its78+pp3	places average value of ax^2 at core-reflector interface; 614 contains 0.5, used frequently in averaging
isc0 a7,ica622 imr470+c imr470+c imr598 its122+c icta7	calculates loss term (i.e., $b\lambda+\lambda\xi cd/\Delta\beta$) times x^2 for core
isc1 a8,ica624 imr470+pp3+c imr470+pp3+c imr598 its122+pp3+c icta8	as above, for reflector

```
ica622          as above, at core-reflector interface
iad624
imr614
imr470+pp3
imr470+pp3
imr598
its122+pp3

ica626          places zero in registers 32, 120
                (626 initially set to zero and left
its32           to act as zero source

its120

ica78           calculates f at x=.05 (must be calculated
idv470          separately to avoid division by zero)
idv470
iad124
imr78
ispz2           z2 is square root subroutine
its168
ics78
idv470
iad168
its168

isc5            calculates remaining 19 values of f
icr19
a9,ics168+c
imr168+c
imr470
idv78+c

iad168+c
iad168+c
imr470
iad80+c
idv470
idv470
iad124+c
iad122+c
imr80+c
ispz2
its170+c
ics80+c
idv470
iad170+c
its170+c
icta9

isc2            calculates f/ax²

g1,ica168+c
idv78+c
its168+c
ictg1
```

isc2 b1,ica166+c iad168+c imr614 imr470 its122+c ictb1	forms areas of f dx/ax^2 prior to trapezoidal integration ($470 = \Delta x$)
isc2 b2,ica122+c iad120+c its122+c ictb2	sums above areas in trapezoidal integration (120 previously set to zero)
isc2 b3,ica122+c ispz1 its122+c ictb3	calculates $\exp(\int_0^x f\, dx/ax^2)$; flad z1 identifies exponential subroutine
isc2 b5,ica34+c imr122+c its34+c ictb5	source term Z times exponential
isc2 b6,ica32+c iad34+c imr614 imr470 its168+c ictb6	forms areas of (Z times exponential) prior to trapezoidal integration
ica626 its166	sets 166 to zero
isc2 b7,ica168+c iad166+c its168+c ictb7	sums areas, in trapezoidal integration, to compute $\int_0^x Z\,(\exp)\, dx/ax^2$
isc0 f1,ica168+c idv122+c idv122+c idv470+c idv470+c idv618 its168+c ictf1	for core region: $\dfrac{\int_0^x Z\,(\exp)\, dx/ax^2}{(\exp)^2\, ax^2}$

```
ica168+pp3
its74
```

sets aside (in 74) $\int_0^x Z\ (\exp)\ dx/ax^2$ at core-reflector interface

```
isc1
g3,ica168+pp3+c
idv122+pp3+c
idv122+pp3+c
idv470+pp3+c
idv470+pp3+c
idv620
its168+pp3+c
ictg3
```

for reflector region:

$$\frac{\int_0^x Z\ (\exp)\ dx/ax^2}{(\exp)^2\ ax^2}$$

```
ica620
iad618
imr614
iex74
idv122+pp3
idv122+pp3
idv470+pp3
idv470+pp3
idv74
its168+pp3
```

as above, at core-reflector interface (using numerator set aside in 74)

```
isc3
icr20
ici19
k1,isc3
ica168+c
icd1
isc2
its34+c
ictk1
```

reverses storage of above (over entire reactor) in order to integrate from 1 to x

```
ica626
its74
```

sets register 74 to zero

```
isc2
b8,ica36+c
iad34+c
imr614
imr470
its78+c
ictb8
```

forms areas prior to integration

```
ica626
its76
```

sets register 76 to zero

isc2 b9,ica78+c iad76+c its78+c ictb9	sums above areas in trapezoidal integration
isc3 icr20 ici19 k2,isc3 ica76+c icd1 isc2 its34+c ictk2 isc2 k3,ica34+c its78+c ictk3	returns above integrals to original order of storage
isc2 c1,ica78+c imr122+c its34+c ictc1	computes flux: $\emptyset = (\exp) \int_1^x \frac{\int_0^x Z\ (\exp)\ dx/ax^2}{(\exp)^2\ ax^2}\ dx$
isc4 ispp1+c p1,ispx1 ispx2 ispx3 ispx4 ispx5 ispa4	counter 4 selects energy group by jump to x1 through x5 (which store $\emptyset_6$ through $\emptyset_2$ and position constants for next computing $\emptyset_5$ through $\emptyset_1$); the isp a4 instruction resets counter 4 and proceeds to the $\emptyset_1$ storage routine
x6,isc2 ica626 c2,iad34+c ictc2 its116	to determine radius criticality calculates an approximation of $\int_0^1 \emptyset_1(x)\ dx$ based on the newly computed $\emptyset_1$ in the registers following 34
isc2 ica626 c4,iad430+c ictc4	as above, based on previously computed (or assumed) $\emptyset_1$
idv116	computes ratio of above integrals γ
isu508	computes $\gamma - 1$
imr--- iad508	multiplies $(\gamma - 1)$ by any desired factor to control rapidity of conversion, plus 1
imr598 its636	computes new λ: $\lambda_2 = \lambda_1\ (1+\{\gamma - 1\}/n)$

```
ica34
isu36               linear extrapolated value of ∅th
iad34               at reactor center to normalize
its122              calculated ∅th

isc2                normalize calculated ∅th to corres-
c6,ica34+c          pond to originally chosen value (in
idv122              register 636) at center
imr638

its430+c            store normalized ∅th in assigned
ictc6               registers

ica636
idv612              calculate Rtotal from λ
ispz2

idv600
iMOA+123.456c       print out Rtotal (sample number given
                    to specify number of significant
                    figures desired)
ica636              determine criticality by comparing
isu598              the per centage change in λ
idv636              with an acceptable error stored in
iad604              register 604; if error is too large--
icpa1               either plus or minus--return to a1
ica598              (the original starting point of the
isu636              program) and recalculate; if error is
idv636              acceptable, continue to next sequence
iad604              of instructions
icpa1

iFOR                specifies format of solution: blocks
+5                  of 20 numbers arranged 5 per line,
+0                  with a tab between numbers
+20

isc2                prints out 20 values of the calculated
c7,ica430+c         flux for each of 6 energy groups con-
iMOA+1.234567f      sidered
ictc7

isc2
c8,ica386+c
iMOA+1.2345f
ictc8
```

```
isc2
c9,ica342+c
iMOA+1.2345f
ictc9

isc2
c10,ica298+c
iMOA+1.2345f
ictc10

isc2
c11,ica254+c
iMOA+1.2345f
ictc11

isc2
c12,ica210+c
iMOA+1.2345f
ictc12
```

calculate the conribution to the total power distribution from each group (fission cross-sections--weighted by $\Delta\phi/c$ for $\phi_{2,3,4,5}$--stored in 520,522,524,526,528)

```
isc0
m6,ica520
imr430+c
its34+c
ica522
imr386+c
iad34+c
its34+c
ica524
imr342+c
iad34+c
its34+c
ica526
imr298+c
iad34+c
its34+c
ica528
imr254+c
iad34+c
its34+c
ictm6
```

linear extrapolation to determine power at center

```
ica34
isu36
iad34
its122
```

```
isc0                    print out power distribution
m7,ica34+c              normalized to unity at center
idv122
iMOA+1.2345f
ictm7

                        insert stopping command (e.g.,
----                    iSTOP or sp26, as discussed
                        on page 54)

x1,isc2                 subroutine to be used after
d1,ica34+c              calculation of Ø6: stores Ø6
its210+c
ictd1                   in assigned registers, places the
ica578                  parameters of the next lower
its618                  energy group in the working
ica580                  registers and calculates the
its620                  source term Z for the next
ica534                  lower group; reenters program
its622                  at entry point set aside on
ica536                  page 56
its624
isc0
d6,ica34+c
imr470+c
imr470+c
imr598
imr554
its34+c
ictd6
ica34+pp3
its602
isc1
e2,ica34+pp3+c
imr470+pp3+c
imr470+pp3+c
imr598
imr556                  (THE FOLLOWING PAGE CONTAINS
its34+pp3+c             FOUR SIMILAR SUBROUTINES WHICH
icte2                   PERFORM THE SAME FUNCTION AFTER
ica554                  CALCULATION OF Ø5, Ø4, Ø3,Ø2)
iad556
imr614
imr602
imr470+pp3
imr470+pp3
imr598

its34+pp3
ispa4
```

```
x2,isc2              x3,isc2              x4,isc2              x5,isc2
d2,ica34+c           d3,ica34+c           d4,ica34+c           d5,ica34+c
its254+c             its298+c             its342+c             its386+c
ictd2                ictd3                ictd4                ictd5
ica582               ica586               ica590               ica594
its618               its618               its618               its618
ica584               ica588               ica592               ica596
its620               its620               its620               its620
ica538               ica542               ica546               ica550
its622               its622               its622               its622
ica540               ica544               ica548               ica552
its624               its624               its624               its624
isc0                 isc0                 isc0                 isc0
d7,ica34+c           d8,ica34+c           d9,ica34+c           e1,ica34+c
imr470+c             imr470+c             imr470+c             imr470+c
imr470+c             imr470+c             imr470+c             imr470+c
imr598               imr598               imr598               imr598
imr558               imr562               imr566               imr570
its34+c              its34+c              its34+c              its34+c
ictd7                ictd8                ictd9                icte1
ica34+pp3            ica34+pp3            ica34+pp3            ica34+pp3
its602               its602               its602               its602
isc1                 isc1                 isc1                 isc1
e3,ica34+pp3+c       e4,ica34+pp3+c       e5,ica34+pp3+c       e6,ica34+pp3+c
imr470+pp3+c         imr470+pp3+c         imr470+pp3+c         imr470+pp3+c
imr470+pp3+c         imr470+pp3+c         imr470+pp3+c         imr470+pp3+c
imr598               imr598               imr598               imr598
imr560               imr564               imr568               imr572
its34+pp3+c          its34+pp3+c          its34+pp3+c          its34+pp3+c
icte3                icte4                icte5                icte6
ica558               ica562               ica566               ica570
iad560               iad564               iad568               iad572
imr614               imr614               imr614               imr614
imr602               imr602               imr602               imr602
imr470+pp3           imr470+pp3           imr470+pp3           imr470+pp3
imr470+pp3           imr470+pp3           imr470+pp3           imr470+pp3
imr598               imr598               imr598               imr598
its34+pp3            its34+pp3            its34+pp3            its34+pp3
ispa4                ispa4                ispa4                ispa4
z1, exponential subroutine (LSR FU1)
z2, square root subroutine (LSR FU 2b)
i START AT 640     (terminates read-in and instructs computer
                    to take first instruction from register 640)
```

Input Data Arrangement

(All even-numbered registers include the following odd numbered one to provide double register storage)

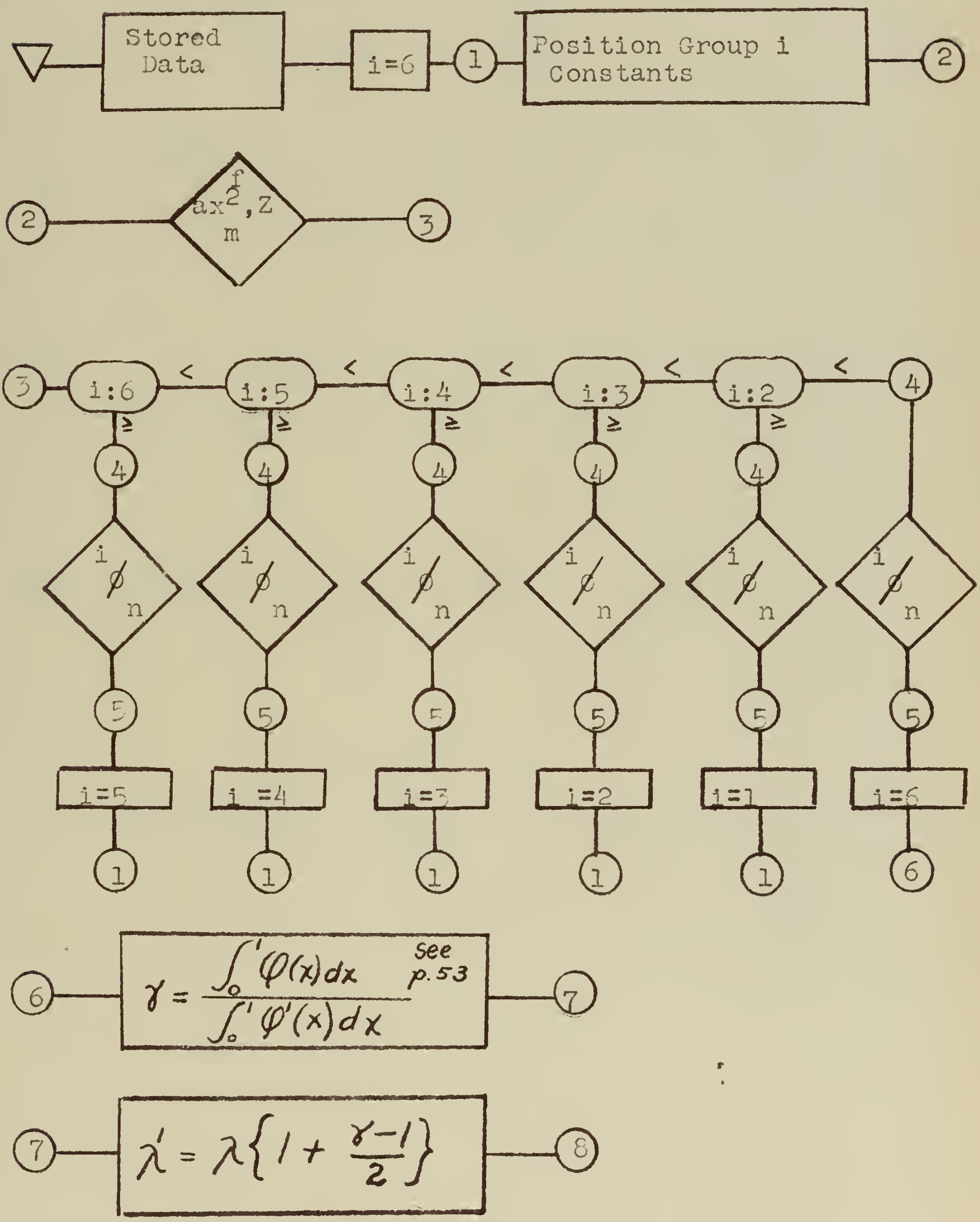
Register(s)	Stored Data		
32-428	Initially set aside as storage during program calculations by inserting zeroes		
430-468	Insert 20 values of assumed thermal flux scaled to unity at center		
470-508	Insert 20 consecutive space points (x=r/R=.05,.10,.15,....95,1.0)		
510	Fission parameters:	$\nu\Sigma_f/\Sigma_{ref}$	5c
512		"	4c
514		"	3c
516		"	2c
518		$(\nu\Sigma_f/\Sigma_{ref})\frac{C}{\Delta\beta}$	1c
520	Fission cross-sections: (for power distribution)	Σ_f	1c
522		$\Sigma_f\Delta\beta/c$	2c
524		"	3c
526		"	4c
528		"	5c
530	Loss parameters: (includes loss to group by both absorption and by slowing down out group, except for thermal group)	$\Sigma_a/\Sigma_{ref}+\frac{\xi C\Sigma_s}{\Delta\beta\Sigma_{ref}}$	6c
532		"	6r
534		"	5c
536		"	5r
538		"	4c
540		"	4r
542		"	3c
544		"	3r
546		"	2c
548		"	2r
550		Σ_a/Σ_{ref}	1c
552		"	1r
554	Input parameters (neutrons slowed into a group from the next higher energy group)	$\xi C\Sigma_s/\Delta\beta\Sigma_{ref}$	6c
556		"	6r
558		"	5c
560		"	5r
562		"	4c
564		"	4r
566		"	3c
568		"	3r
570		$\xi\Sigma_s/\Sigma_{ref}$	2c
572		"	2r

574	Dimensionless transport parameters $(\Sigma \text{ref}/\Sigma \text{tr}_i)$	6core
576		6refl
578		5c
580		5r
582		4c
584		4r
586		3c
588		3r
590		2c
592		2r
594		1c
596		1r
598	For storage of λ (initially zero)	
600	(Σref) for computing radius from (λ)	
602	Temporary storage for values at core-reflector interface	
604	Insert allowed error in (λ): ϵ	
606	Storage for exponential subroutine	
608-610	Storage for square root subroutine	
612	Insert +3. to compute r from (λ)	
614	Insert +.5 for linear averaging	
616	unused	
618-622	Initially zero-storage for parameters of each group during main program	
624	Insert +0. for zero source	
626-634	unused	
636	Insert assumed (λ)	
638	Insert scale factor to determine value of centerline thermal flux (such as +1. for thermal reactors, +.1 for intermediate, +.01 for fast)	
640	(Start of instructions)	

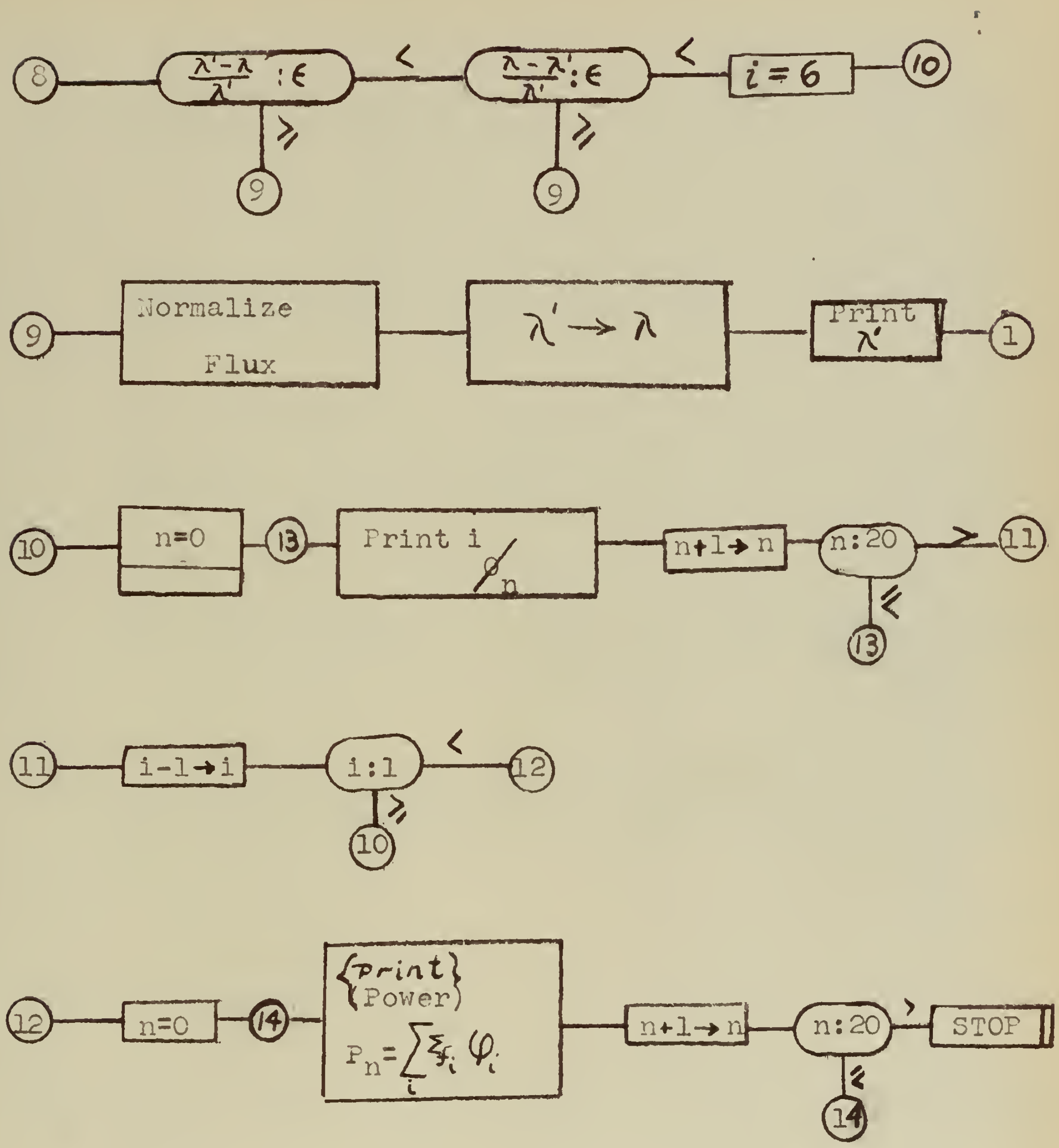
FLOW SHEET FOR SIX GROUPS

(Superscript i denotes energy group, from 1 to 6)

(Subscript n denotes space point, from 1 to 20)

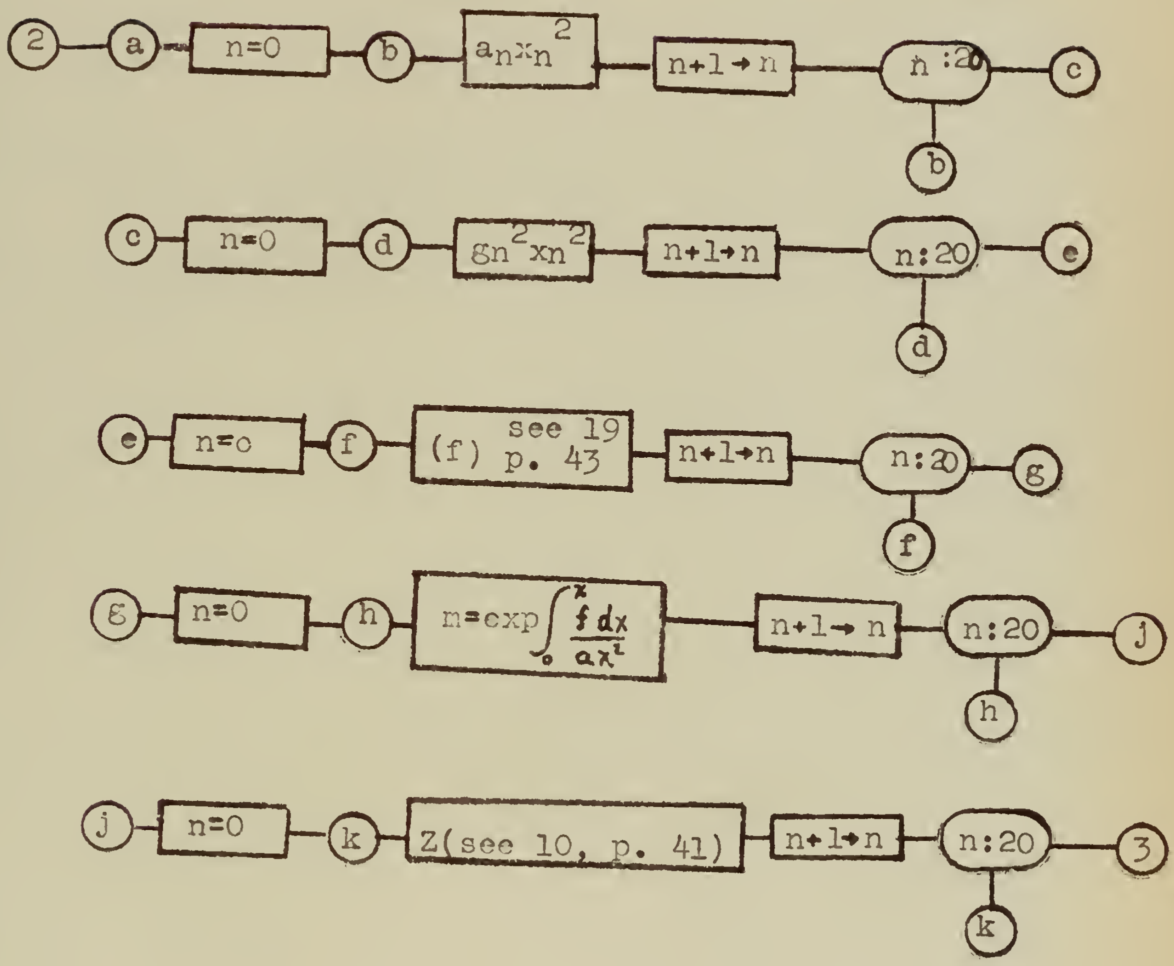

Stored Data
i=6
1
Position Group i Constants
2
2
ax^2, Z f m
3
3
i:6
i:5
i:4
i:3
i:2
<
≥
4
ϕ^i_n
5
i=5
i =4
i=3
i=2
i=1
i=6
1
6
6
$\gamma = \frac{\int_0^1 \varphi(x)dx}{\int_0^1 \varphi'(x)dx}$
see p.53
7
7
$\lambda' = \lambda\left\{1 + \frac{\gamma - 1}{2}\right\}$
8

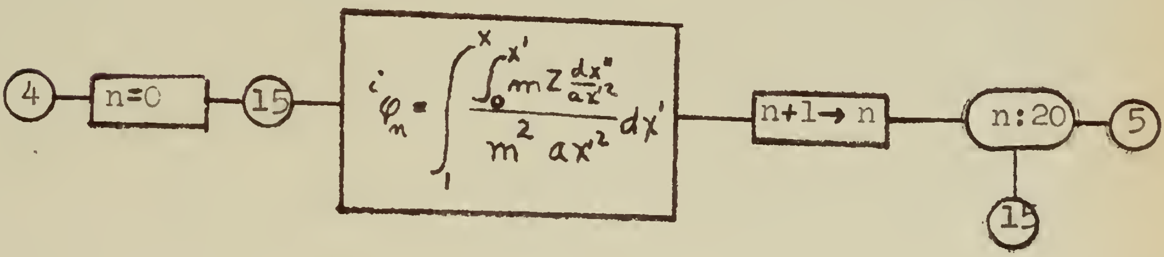
FLOW SHEET FOR SIX GROUPS (Continued)


8
$\frac{\lambda' - \lambda}{\lambda'} : \epsilon$
<
$\geqslant$
9
$\frac{\lambda - \lambda'}{\lambda'} : \epsilon$
<
$\geqslant$
9
$i = 6$
10
9
Normalize Flux
$\lambda' \rightarrow \lambda$
Print λ'
1
10
n=0
13
Print i ϕ_n
n+1→n
n:20
>
11
<
13
11
i-1→i
i:1
<
12
$\geqslant$
10
12
n=0
14
{Print Power}
$P_n = \sum_i \Sigma_{f_i} \varphi_i$
n+1→n
n:20
>
STOP
<
14

FLOW SHEET FOR SIX GROUPS (Continued)

SUBROUTINES


2
a
n=0
b
$a_n x_n^2$
n+1→n
n:20
c
b
c
n=0
d
$g_n^2 x_n^2$
n+1→n
n:20
e
d
e
n=0
f
(f) see 19 p. 43
n+1→n
n:20
g
f
g
n=0
h
$n = \exp\int_0^x \frac{f\,dx}{ax^2}$
n+1→n
n:20
j
h
j
n=0
k
Z(see 10, p. 41)
n+1→n
n:20
3
k


4
n=0
15
$\varphi_n = \int_1^x \frac{\int_0^{x'} mZ \frac{dx''}{ax''^2}}{m^2\, ax'^2}\, dx'$
n+1→n
n:20
5
15

CHAPTER III

RESULTS

I. Reactor Compositions Studied

a. Core Materials. In order to simplify the problem, a choice was made as to the type of fuel element, coolant and moderator. The choice was based on a considered adaptability to high output mobile reactors. High power requirements, coupled with a need for light weight and small size, dictated the use of highly enriched uranium and cladding materials which would withstand high temperatures.

The fuel element was a dispersal element of uranium dioxide in a matrix of stainless steel. Some calculations have been made on this type of fuel element[1], and good high temperature properties are envisioned. The amount of uranium dioxide in the steel matrix is kept low in order to minimize radiation damage and to improve heat transfer properties. A similar fuel element, with the steel replaced by aluminum, was operated at the Geneva conference on the peaceful uses of atomic energy.

The uranium considered is enriched to 90% U-235. The density[2] of UO_2 is 10.95 gm/cc.

Stainless steel is also used as the cladding for the fuel element. Its choice is dictated by the fact[3] that it will withstand sodium up to about 1600°F. Its absorption

cross section presents little problem with the fuel enrichment considered here.

Beryllium is chosen as moderator because of its good moderating properties, light weight, and good thermal properties. It is protected from the sodium coolant by stainless steel, since beryllium withstands sodium only to about[3] 1000°F and higher temperatures than this are considered.

Choice of sodium as the coolant is based on its good heat removal characteristics and high boiling point. A high boiling point eliminates the need to pressurize the core.

The reflector is considered to be composed of various proportions of Be, Na, and stainless steel. Reflector cooling is necessary since some five per cent[4] of the heat may be liberated in the reflector. This amount of heat will be considerable for high output reactors. Stainless steel is required, as in the core, to protect the beryllium from the corrosive action of high temperature sodium, and for structural purposes.

With a basic reactor chosen, variations in the proportions of the materials were studied in order to determine their effect on the power distribution. Table III of Appendix B summarizes reactor compositions studied.

The effect of reactor poisons was not considered. All of the reactors were assumed to be homogeneous and in the steady state.

b. Cross Sections of Reactor Materials. The cross sections of the materials considered are listed in Tables I and II of Appendix B. They are based on data from "Neutron Cross Sections," BNL 325 (August, 1955, Geneva Supplement).

c. Reactor Configuration. For reasons of mathematical simplicity the reactor considered is spherical, with a reflector of uniform thickness. Because of angular symmetry the spherical reactor may be resolved to a one-dimensional problem. Cylindrical or rectangular geometry would, on the other hand, require two- or three-dimensional analysis. While it may be argued that spherical reactors will not find extensive use, the results from an analysis of spherical reactors can be extrapolated to cover reactors of other configurations.

The magnitude of the multi-dimensional problem is such that simplification to a single dimension is made frequently. Other than the spherical case considered here, the only one-dimensional geometry is that of the infinite slab. Spherical geometry is superior, since it gives results which may be applied to reactors of other configurations whose dimensions along various axes are not greatly different.

d. Fuel Concentration. The proportion of fuel in the reactor has an important effect on the radius of the critical reactor. It is also related closely to the power

level.

For the mobile power reactor envisioned here, there are certain restrictions on the size which must be considered. It cannot be so large that its mobility is questionable, and yet it must be large enough so that enough coolant may be pumped through to remove the required heat.

From elementary heat transfer considerations (Appendix A), a core radius of from two to four feet was found to be sufficient for heat removal, and since this size appeared reasonable for mobile applications, the amount of fuel was adjusted in order to give a radius within these dimensions.

e. Moderator Proportion. The amount of moderator considered varied from zero to sixty per cent of the core volume. This had the effect of changing the reactor from an essentially thermal one to an intermediate one, in which the greater part of the fissions take place at energies appreciably above thermal.

f. Reflector Composition. It was determined that as little as two per cent of the reflector volume was required to be sodium in order to carry off heat generated within the reflector. The amount of stainless steel was arbitrarily taken as five per cent.

By assigning the remainder of the reflector volume to the beryllium, it was found that the neutrons returned to

the core were largely thermal. In order to decrease the number of moderated neutrons reflected back to the core, additional sodium was introduced into the reflector and the beryllium decreased, so that the percentage of beryllium considered ranged from a low of 55% to a high (one run only) of 95%.

II. Effect of Various Parameters.

a. Fuel Concentration. Apart from its effect on the critical radius of a reactor, the fuel concentration plays an important part in determining the energy spectrum of neutrons in the reactor. Neutrons are born fast and, during the slowing down process, may be absorbed by reactor materials. Absorption during slowing down increases the number which must be born fast in order that enough remain after slowing down to cause further fissions and perpetuate the chain reaction.

An examination of the cross sections of the materials considered here shows that the predominant absorber of neutrons having energies greater than thermal is the uranium. Increasing the fuel thus increases the flux in higher energy groups.

Actually there are three things which may happen to a neutron while slowing down: it may be absorbed; it may be lost through leakage; or it may undergo scattering collisions and be slowed down. The fuel concentration is so small that it can have little effect on the slowing down power.

The fuel's important effect is on absorption tendencies and, as a consequence, on the leakage, since an increase in fast flux with more fuel increases leakage.

It is important to note that of the neutrons absorbed by the uranium during the slowing down process, roughly four-fifths cause fission of the absorbing nucleus. Thus more fuel has the added effect of changing an originally thermal reactor toward a reactor in which fissions at energies above thermal predominate.

Figure 1 demonstrates the above effect of fuel concentration on the ratio of fast to thermal flux. The ratio, calculated at the reactor center, is plotted versus fuel concentration for twenty different reactors all having at least 30% beryllium in the core.

It should be pointed out that the ratio of fast to slow flux has an important effect on the ability of the reflector to assist in levelling the power distribution. A greater number of fast neutrons leaking into the reflector increases the number of slowed neutrons which the reflector can return to the core to compensate for slow leakage, which would otherwise depress the power at the edge of the core.

b. Moderator Concentration. As mentioned above, the ratio of the fast to the slow flux depends on leakage, absorption, and slowing down. Changing the moderator proportion in the core affects the first and last terms. For simplicity of discussion its effect on leakage may be

neglected, if the central regions of the reactor are considered.

In comparing the remaining two terms--absorption and slowing down--it is important to note that the criterion for comparison is the ratio of either to the sum of the two. That is, the total probability of one or the other happening (excluding leakage) must be unity. For this reason, decreasing one has the effect of increasing the other.

The beryllium moderator is the predominant contributor to the slowing down term by virtue of its high value of ξ. Any decrease in beryllium thus tends to enhance the absorption in uranium during slowing down. Thus the ratio of fast to slow flux varies inversely with the beryllium concentration as shown in Figure 2 where ϕ_{fast}/ϕ_{th} is plotted as a function of per cent beryllium for reactors of various compositions.

c. Effect of fuel and moderator concentrations On Power Distribution

As discussed above, either more fuel or less beryllium increases the ratio of fast to slow flux and thus raises the ability of the reflector to return slowed neutrons to the core and assist in levelling the power distribution. Fig. 3 shows this effect for various beryllium concentrations in an otherwise identical reactor. As the beryllium decreases, the power near the core-reflector boundary rises.

The uniformity of power distribution, measured by the ratio of maximum to average power, depends on the point at which P_{max} occurs. This may be either at the center, the edge, or, ideally, both simultaneously. That is, P_{edge} should be adjusted to equal the center power in order to increase P_{avg} as much as possible without simultaneously raising P_{max}. As an indication of this, the ratio of P_{max}/P_{avg} in Fig. 3 decreases to 1.16(as beryllium decreases from 30 to 20 per cent), then rises to 1.62 as beryllium is further decreased. In the latter case the edge power is limiting.

Fig. 4 shows a similar effect on the power distribution for various fuel concentrations. Figs. 4a and 4b indicate flux variations for various fuel concentrations. Fig. 4b shows that with more fuel the greater fast flux results in a significant number of neutrons being returned to the core before being completely moderated. These aid in levelling the power distribution.

Also in Fig. 4b (0.5% fuel) the large ratio of fast to thermal flux results in some 18% of fissions taking place in the two higher energy groups, as contrasted with only 9% for the reactor of Fig. 4a (0.3% fuel).

d. Reflector Composition

The effect of varying the proportions of the three reflector components was investigated. In general, for the reflector compositions studied, beryllium makes the largest contribution to the slowing down term and the smallest to

the absorption. Therefore, decreasing the reflector's beryllium concentration effectively increases the absorption tendencies while decreasing the moderation. Reflectors with less beryllium thus return fewer slowed neutrons to the core.

Fig. 5, which shows the power distributions for three reactors with identical cores but with different proportions of beryllium in the reflector, demonstrates this effect. The rise in power occasioned by neutrons returned from the reflector is greatest for the reactor having the most beryllium in the reflector, least for the reactor with least reflector beryllium.

e. Reflector Thickness

Figs. 6, 7, and 8 show the spatial distribution of flux for reactors with reflector thicknesses of 10, 20, and 50 cm respectively. Since fast neutrons which leak into the reflector are subject to a greater chance of slowing down in a thick reflector, the thermal flux rises more in such a reflector. In turn, this causes a power rise near the core-reflector interface, since more thermal neutrons are returned to the core.

This provides a means of increasing the thermal flux in the outer core region where the power normally is less than at the reactor center. By a proper choice of reflector this increased thermal flux in the outer core region can result in a power level as high as the power

at the reactor center. This represents the closest possible approach to a uniform power distribution for a given homogeneous reactor, since the power distribution sags between these two maximums.

Fig. 9 shows the effect of varying the reflector thickness on the power distribution for a given core. A 10 cm reflector gives a power peak at the reactor center. Somewhere between 10 and 20 cm reflector thickness the optimum power distribution for this reactor occurs. For 20 and 50 cm reflector thicknesses a power peak occurs at the surface of the spherical core. With increasing reflector thickness a limiting value of P_{max}/P_{avg} is approached asymptotically. A plot of the neutron flux in Fig. 8 for the 50 cm reflector shows that the flux in all energy groups approaches zero asymptotically to the axis. Since the current density $J = -Dd(NV)/dr$ (as given in Chapter I), a nearly zero slope to the flux curve means that there are very few neutrons escaping from the outer reflector surface. This indicates that a further increase in reflector thickness would have little effect on the power distribution, so that this is essentially an infinite reflector.

The positive slope (at the core-reflector interface) to the thermal flux curve means that J is negative, indicating a net flow of neutrons from the reflector into the core. It is noted that with reflectors of 10 and 20 cm thickness there is a net leakage of group III neutrons

into the reflector. However, with a 50 cm thick reflector there is a positive slope to the group III flux at the interface, indicating a net return of group III neutrons from the reflector to the core. This tends further to increase the power rise in the outer core regions.

The significance of fissions in group III can be further investigated. Figs. 4a,4b, 6, 7, and 8 show flux plots for different reactors. Histograms in these figures show that approximately 60% of the total fissions take place in energy group III. Thus, one would expect that the spatial power distribution would follow the core flux plot of group III quite closely. An examination of the figures mentioned shows that the power distribution does follow the group III flux very closely except in the outer core regions where the thermal flux begins to rise. At this point the power curve also exhibits an upward trend. This reflects the fact that the thermal neutron fission cross section is approximately three and a half times that of the average group III cross section, so that a small increase in thermal flux has great significance.

Another trend which can be noted from the figures cited is that the group III flux distribution sags continually as distance from the center increases. Thus power which depends primarily on group III will sag also, even though the thermal flux remains relatively flat. If more moderator is added so that the reactor approaches more closely a thermal reactor, the sag in the thermal flux in the outer regions will not be compensated for

by a return of reflector moderated neutrons, since, with relatively few fast neutrons present there will be little fast leakage input to the reflector. Fig. 4a illustrates this trend.

f. Effect of Varying Na/Fe Ratio

In Fig. 11 is plotted power distributions for three reactors identical except for the sodium and stainless steel content of the core. The curves are very similar except for the outer core region.

Because Fe has a higher absorption cross section it is to be expected that increasing the core Fe content will tend to raise the fast flux somewhat, thereby increasing fast leakage and the number of slowed neutrons returned from the reflector. This point is supported by the plot of run (11) which has the highest value of power at the core-reflector interface, and the highest Fe content.

An additional effect is the variation in diffusion coefficient with varying Na/Fe ratio in the core. Since Fe has a higher scattering cross section (by about a factor of three) than sodium, and a higher atomic density (by a similar factor) a greater Fe content tends to decrease D and thereby decrease leakage. It is thought that this partially explains the slight difference between the plots of runs (1) and (10) as shown in Fig. 11. Run (1), with less iron and more leakage experiences a slightly greater

rise near the core-reflector boundary.

Since the two effects mentioned are in opposition the above discussion is rather hypothetical. In general, varying the Na/Fe ratio has little effect on the overall power distribution.

g. Optimum Power Distribution

In Fig. 10 are plotted two optimum power distributions obtained. Shapes of the curves, even though quite different core compositions were considered, are practically identical. This tends to indicate that a limiting value of power uniformity has been met. In other words, the power sag observed evidentally cannot be further lessened for the reactor type and size considered.

The two reactors considered in Fig. 10 have critical radii of 42 cm and 60 cm. Since reactor size affects the proportion of the core which may be affected by reflector moderated neutrons, it is thought that size will play an important part in determining uniformity of power. A reactor with a smaller core will have a larger proportion of volume which experiences a power rise due to reflector moderated neutrons. However it can be seen in Fig. 10 that decreasing the size from 60 to 42 cm has essentially no effect on power distribution.

The two reactors considered in Fig. 10 have different material compositions and different critical radii. They are representative of the class of reactors investigated. It can be concluded, therefore, that in the range of size and composition investigated the optimum

power curve has a characteristic shape which is that shown in Fig. 10. The ratio of P_{max}/P_{avg} for this curve is approximately 1.16.

h. Comparison of Four and Six Group Methods

Fig. 13 gives a comparison of calculations by four group and six group methods for the same reactor. In general the results are in good agreement, since the critical radii differ by less than four per cent. The ratio of fast to thermal flux was 3.4 for four groups and 4.7 for six groups. These values are of the same order of magnitude.

The plot of spatial power distribution in the core shown in Fig. 13 gives different values of P_{max}/P_{avg} for the two methods, although in general the curves have similar characteristics. Better agreement in power distribution between methods seemed to exist for reactors having other compositions, although other runs on identical reactors were not made due to time limitations. Since both methods use the age diffusion equation (with minor differences which are covered in the discussion of results) the results should be very similar. Experimental results for Na, Be, Fe intermediate reactors were not available for a check on the accuracy of the results.

REFERENCES

1. C. E. Weber and H. H. Hirsch: "International Conference on the Peaceful Uses of Atomic Energy", Volume 9, Paper 561.

2. J. R. Johnson and C. E. Curtis: "International Conference on the Peaceful Uses of Atomic Energy", Volume 9, Paper 559.

3. Reactor Handbook (Engineering), AECD-3646, U. S. Atomic Energy Commission, May 1955, (298).

4. S. Glasstone: "Principles of Nuclear Reactor Engineering", D. Van Nostrand Co., New York, 1955 (646).

DISCUSSION OF RESULTS

The principal results of this study have been the determination of critical radius, neutron flux and fission power distribution, and the effect upon these quantities of varying the material compositions of the core and reflector as well as varying the reflector thickness.

The method of attack involved modifying the age diffusion equation so as to permit solutions to be obtained by use of Whirlwind I digital computer. One method was developed in Chapter I which used a finite difference approximation to the age diffusion equation with neutrons divided into four energy groups. A second method was developed in Chapter II which used a method of numerical integration with neutrons divided into six energy groups.

The results of calculations based upon identical reactors using the two different methods were in substantial agreement. However, minor differences in spatial power distributions and critical radius were noted for solutions by the two methods.

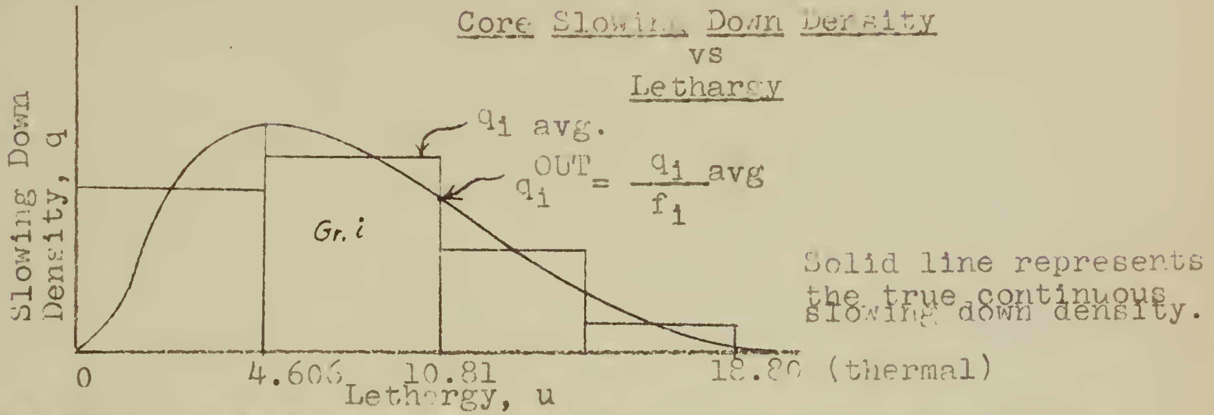
The explanation for differences in the radius and power distribution given by the two methods is thought to be the result of differences in calculated average cross sections for the various energy groups. The energy groups are so wide that it is necessary to obtain an average cross section and diffusion coefficient for each energy group. The process of averaging could result in incompatibilities

between average material properties calculated for the two methods, particularly in regions where the cross section changes rapidly with energy, as is the case for the scattering resonance of sodium near 2700 ev.

Another major difference between the methods involved the determination of the elastic scattering input from energy group i to the next lower energy group i+1. In the four-group method a factor f was chosen such that $q_{i_{avg}} = f\ q_i^{out}$. The solution was not sensitive to the value chosen for f as shown by successive runs using improved values of f. Chapter I (page 4) discusses the manner in which f can be improved from run to run on a trial and error basis. The use of the f factor gives good results for wide energy groups. The disadvantage of having to estimate f can be overcome by considering a greater number of energy groups where the variation of the slowing down density, q, over the group can be considered to be linear. In this case sufficient accuracy could be obtained by the assumption that $q_{i_{avg}} = \frac{1}{2}(q_i^{in} + q_i^{out})$.

In the six group method, however, the assumption is that the elastic scattering out of an energy group is equal to the average value of the slowing down density in an energy group, $q_{i_{avg}} = q_i^{out}$. This is equivalent to assuming that the f factor equals unity. These two different assumptions concerning the elastic scattering out of an energy group probably account principally for any differences between results obtained by the two

methods. This difference can be illustrated by the following figure:



Solid line represents the true continuous slowing down density.

The solid line represents the true distribution of slowing down density. The slowing down density, q, increases for low lethargy values corresponding to the birth energies (10 to 0.1 Mev) of fission neutrons. For lethargies greater than the birth lethargy, q decreases due to absorption and leakage. For the relatively wide lethargy groups shown it can be seen that the assumption used with the six group method that $q_i^{avg} = q_i^{out}$ is not nearly as true as the representation of the true continuous slowing down curve by the assumption that $q_i^{avg} = f\ q_i^{out}$ with a value of f chosen so as to approximate the true curve as closely as possible.

The disadvantages of using a relatively small number of energy groups are as follows: (1) the true elastic scattering out of an energy group is difficult to estimate correctly; (2) determining average neutron cross sections for a particular lethargy group introduces errors, particularly when cross sections vary rapidly with energy. Considering a larger number of groups greatly reduces these problems.

The following advantages result from using a

small number of energy groups: (1) it is possible to neglect elastic scattering across groups (for the materials considered); (2) the energy spectrum of fission neutrons need not be known with great accuracy if group widths are such that all fission neutrons can be considered to be born in one or two energy groups.

The four group program is arranged to include the effects of inelastic scattering. However, in this investigation inelastic scattering is neglected, since it is considered to be of only minor importance for group widths and materials considered.

The plots of neutron flux and power distribution gave consistent results indicating qualitative trends as parameters were varied. The parameters considered were: (1) fuel concentration; (2) reflector composition; (3) Na/Be ratio; (4) Na/Fe ratio; and (5) reflector thickness. An effort was made to choose reactors which would have the maximum power occurring equally at the reactor edge and center to get the smallest value of P_{max}/P_{avg} (ideally equal to unity). The best value that could be obtained was 1.16 for run (8). Fig. 11 shows plots for two optimum power distributions. Comparison of these two curves with other power distributions near the optimum seems to indicate that the general shape of the curve cannot be changed very much for the range of parameters investigated. However, these results would not apply to reactors having a much larger or

smaller critical radius since the shape of the optimum power curve would very likely be considerably changed with marked changes in reactor size.

A ratio of 1.16 for P_{max}/P_{avg} was the best value obtained for the reactor size and type discussed.

The two programs which were developed for use with the Whirlwind I digital computer operated very satisfactorily, and it is hoped that other investigators will find these workable programs to be helpful. Both programs were characterized by rapid convergence to the answer, averaging only about four or five iterations before the convergence criteria were met.

For further work on the subject of uniform power distribution, it is suggested that multi-region reactors be investigated. The Whirlwind programs developed in Chapters I and II can be modified so as to apply to reactors having several regions of uniform material properties. The programs would lengthen considerably, however, and it probably would be necessary to use auxiliary Whirlwind storage.

It would be of interest to compare the results of this theoretical investigation with experimental results in order to determine how closely a finite difference or numerical integration approximation to the age diffusion equation will represent reality.

An extension of this study to investigate reactors

of a different range in size using other moderators and coolants would supplement the results given here.

SUMMARY

It was the objective of this thesis to study the practicability of achieving a uniform power distribution in a homogeneous nuclear reactor, through return of moderated neutrons from a reflector to the core to compensate for leakage. Multi-group methods of solution of neutron diffusion equations were developed. While it was impossible to compare the theoretical results obtained with experimantal data, because of lack of such data, the results appear reasonable in that they demonstrate trends which are in agreement with physical considerations.

Studies were based on a single basic reactor chosen because of suitability to high output mobile applications.

It was found, for the reactor type and size studied that, while complete uniformity of power could not be achieved, nevertheless it was possible to approach uniformity quite closely. The best value obtained for P_{max}/P_{avg} was 1.16.

FIG. 1.

EFFECT OF FUEL CONCENTRATION
ON RATIO OF ϕ_{fast}/ϕ_{th} AT REACTOR CENTER

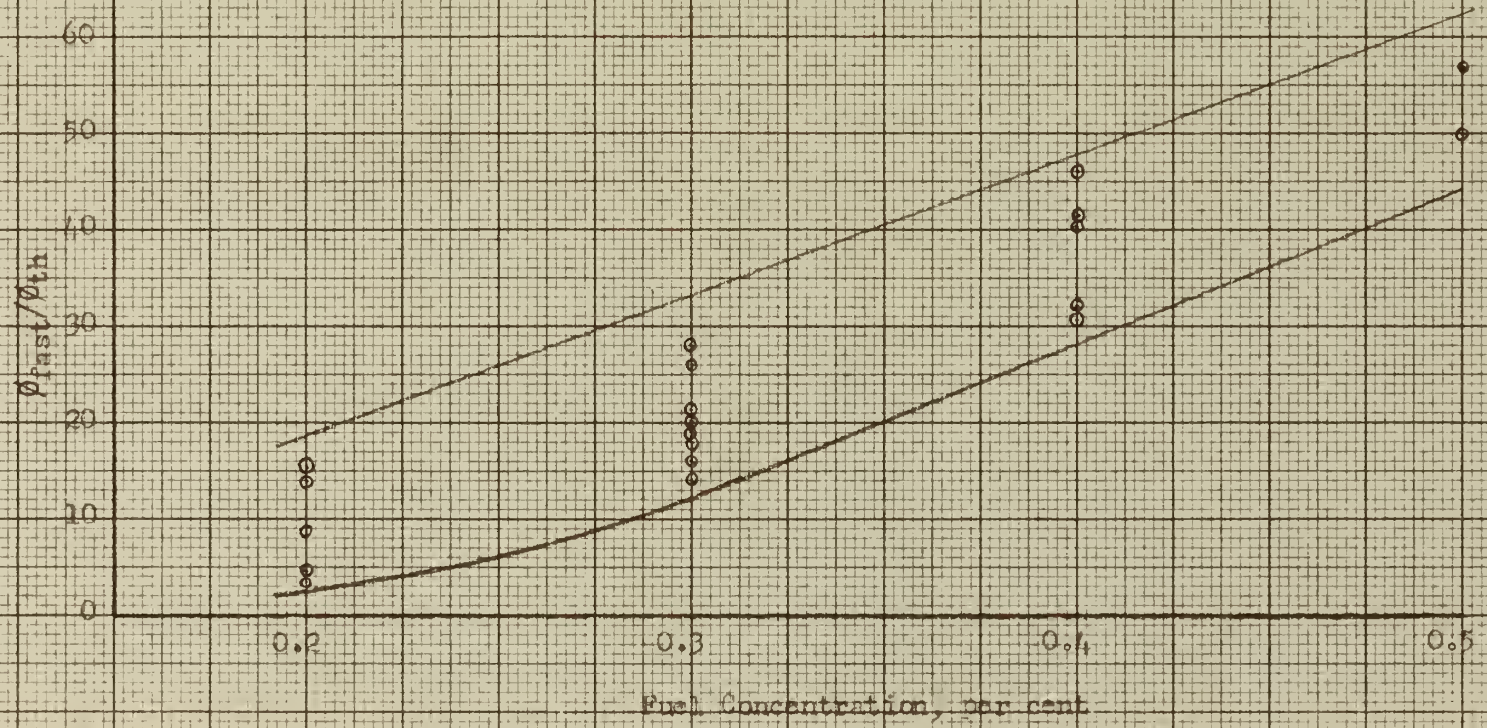
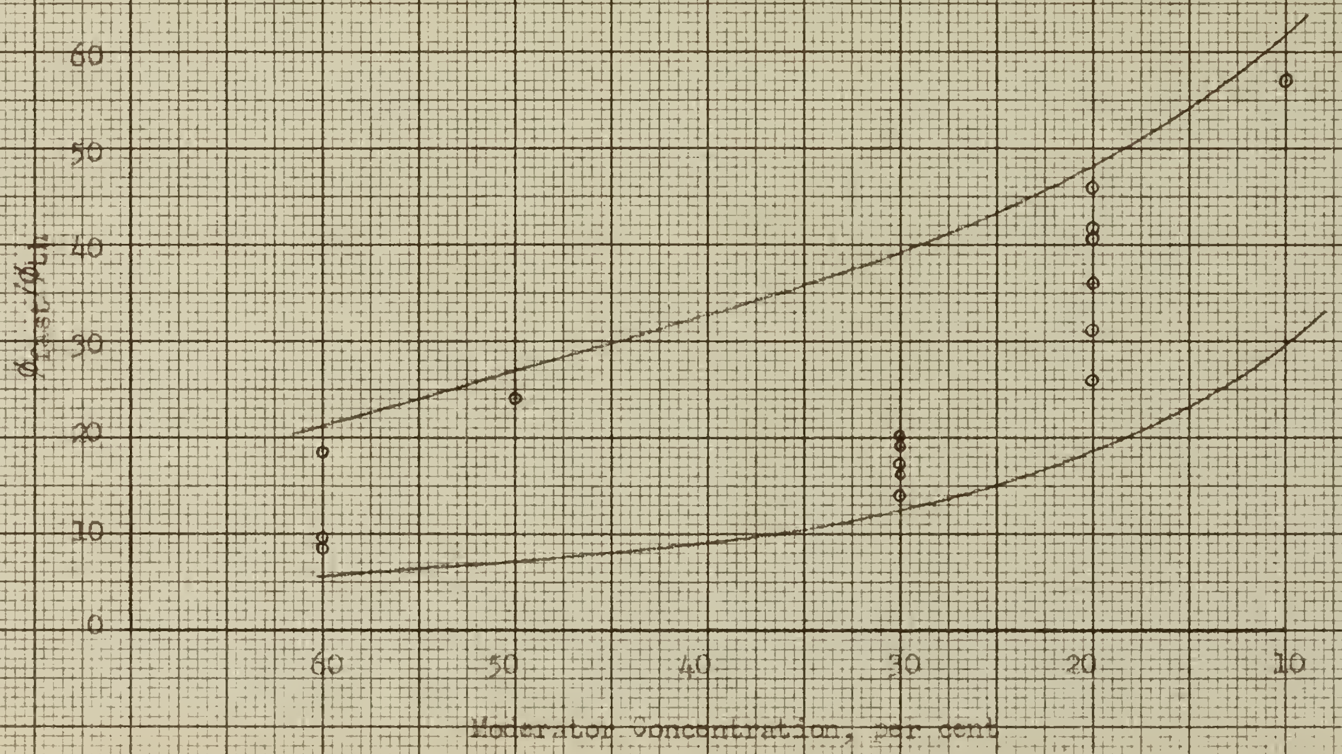


FIG. 2.

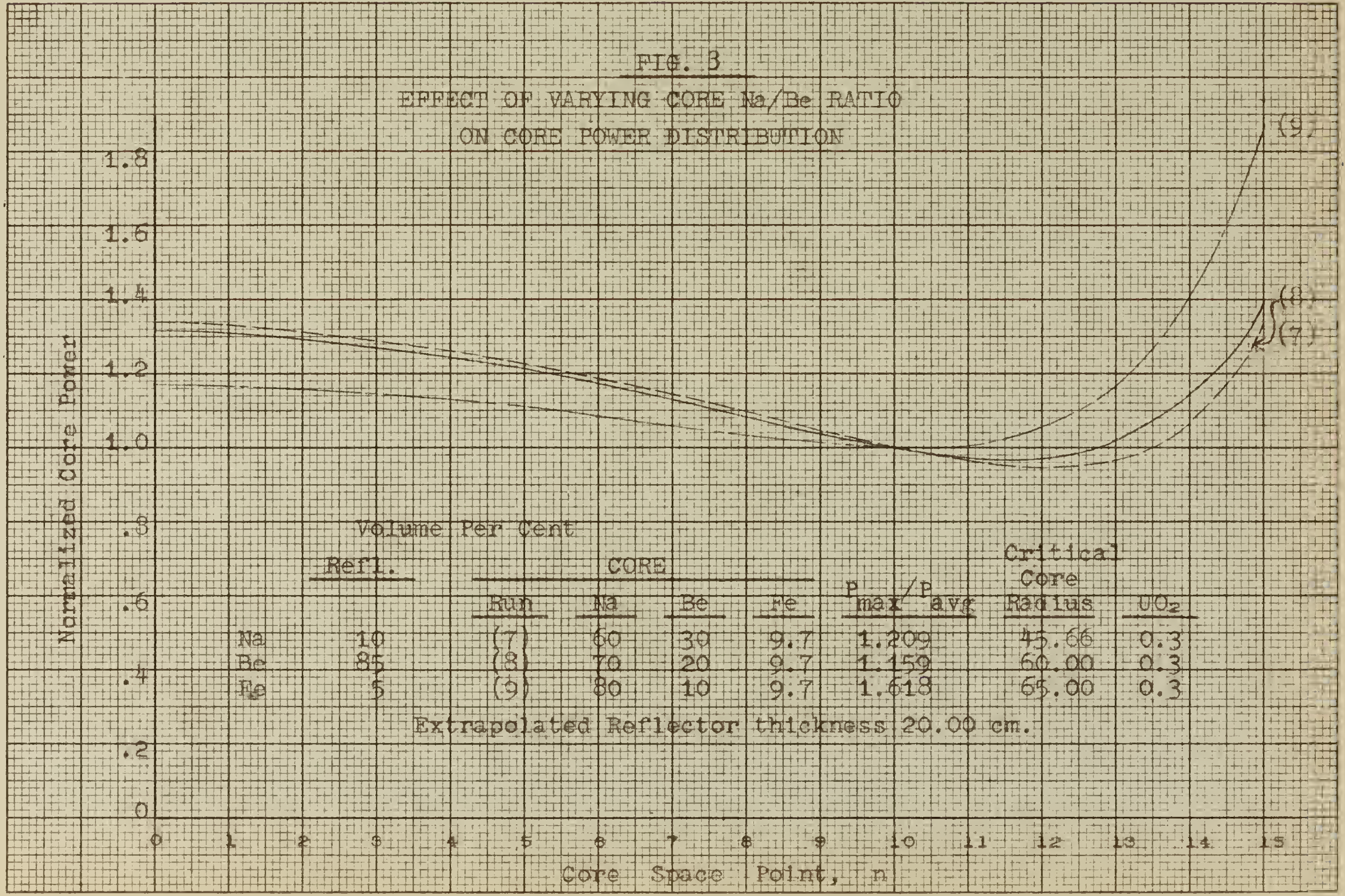
EFFECT OF MODERATOR CONCENTRATION ON RATIO OF ϕ_{fast}/ϕ_{th} AT REACTOR CENTER



(abscissa reversed to show trend as in Fig. 1)

FIG. 3

EFFECT OF VARYING CORE Na/Be RATIO
ON CORE POWER DISTRIBUTION



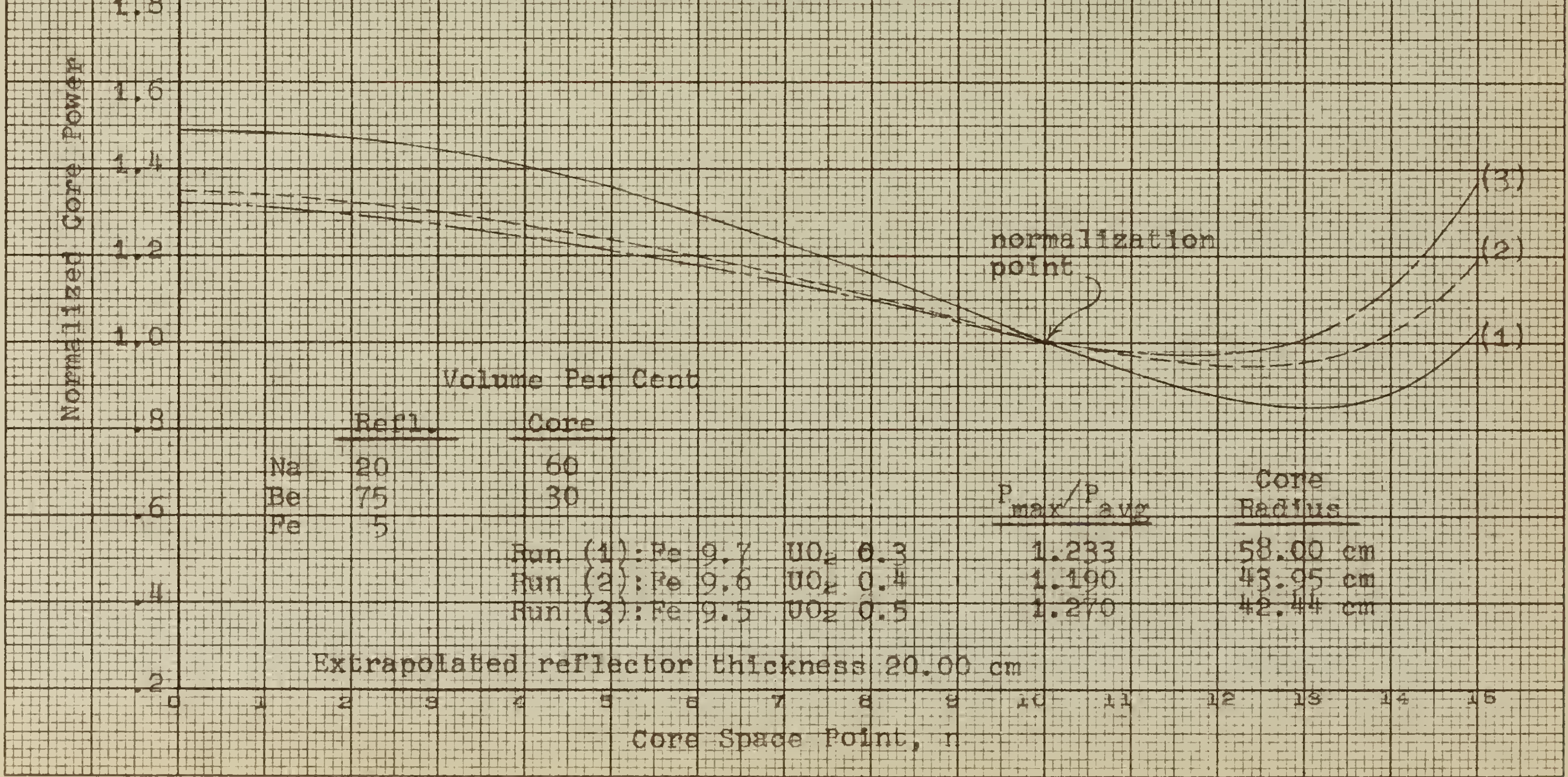
Volume Per Cent

Refl.		CORE					Critical Core	
		Run	Na	Be	Fe	P_{max}/P_{avg}	Radius	UO_2
Na	10	(7)	60	30	9.7	1.209	45.66	0.3
Be	85	(8)	70	20	9.7	1.159	60.00	0.3
Fe	5	(9)	80	10	9.7	1.618	65.00	0.3

Extrapolated Reflector thickness 20.00 cm.

FIG. 4

EFFECT OF VARYING FUEL CONCENTRARION ON CORE POWER DISTRIBUTION



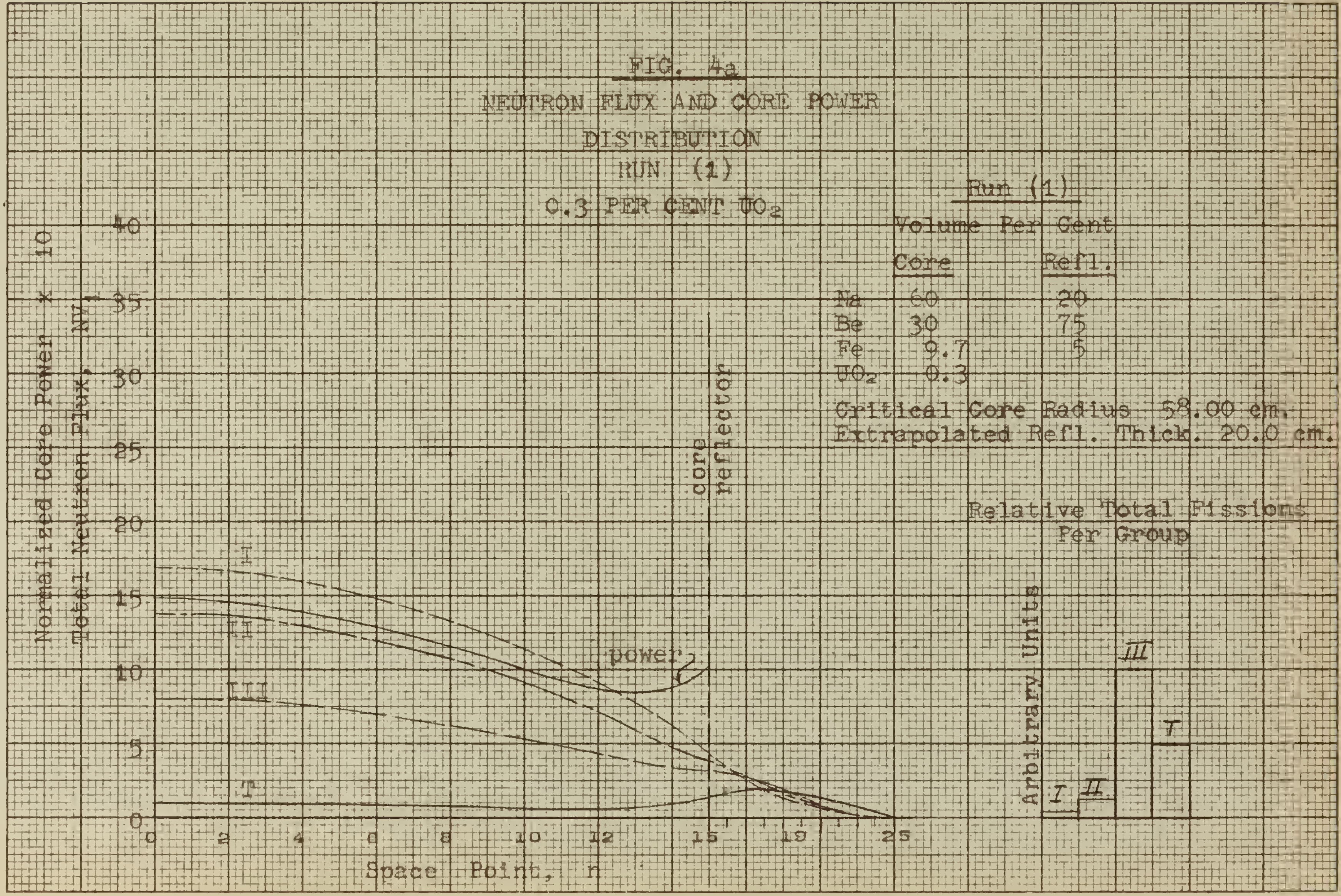


FIG. 4a

NEUTRON FLUX AND CORE POWER DISTRIBUTION

RUN (1)

0.3 PER CENT UO_2

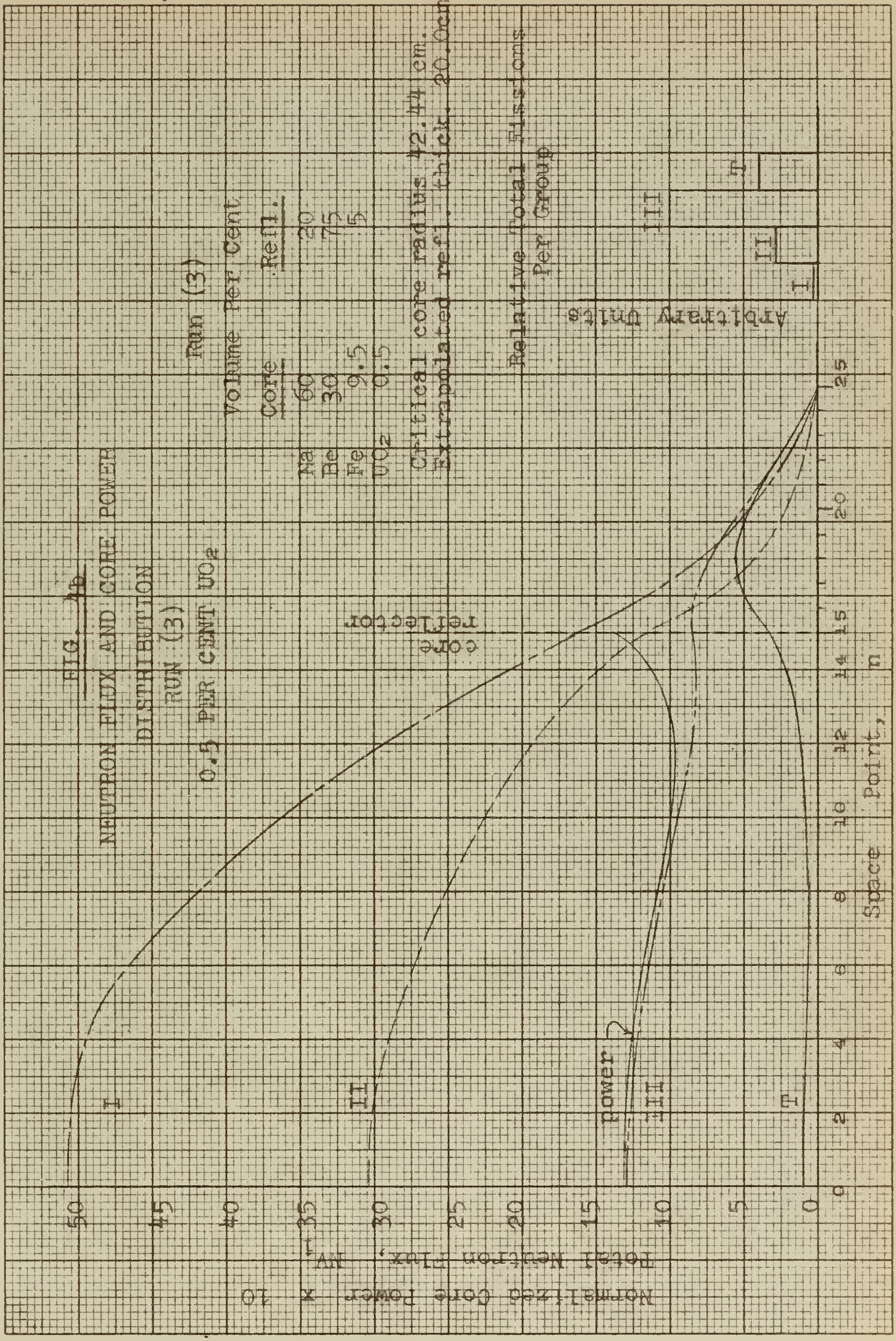


FIG. 4b

NEUTRON FLUX AND CORE POWER DISTRIBUTION

RUN (3)

0.5 PER CENT UO_2

Run (3)

Volume Per Cent

	Core	Refl.
Na	60	20
Be	30	75
Fe	9.5	5
UO_2	0.5	

Critical core radius 42.44 cm.
Extrapolated refl. thick. 20.0cm

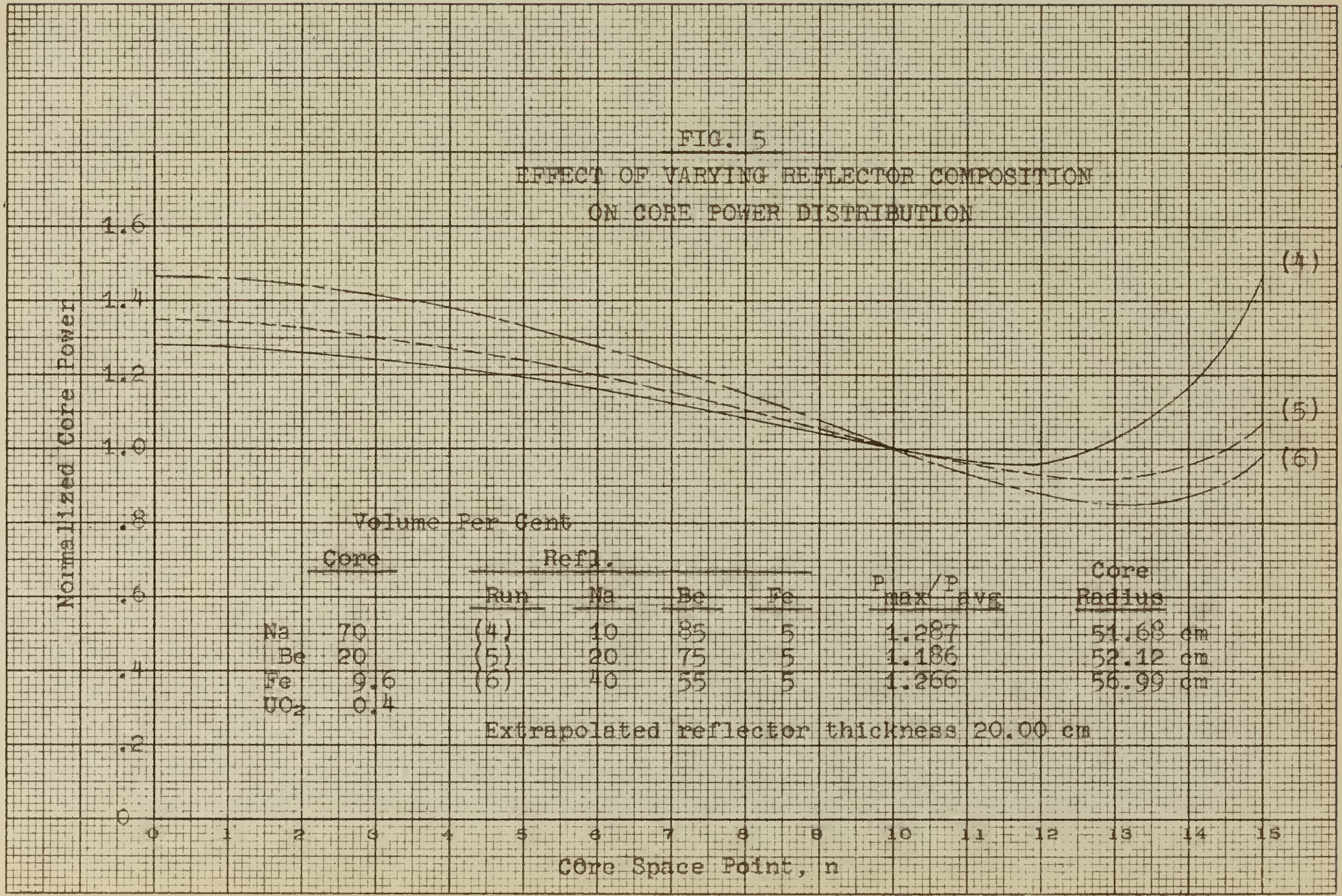


FIG. 5

EFFECT OF VARYING REFLECTOR COMPOSITION ON CORE POWER DISTRIBUTION

Volume Per Cent

Core	
Na	70
Be	20
Fe	9.6
UO_2	0.4

	Refl.				
Run	Na	Be	Fe	P_{max}/P_{avg}	Core Radius
(4)	10	85	5	1.287	51.68 cm
(5)	20	75	5	1.186	52.12 cm
(6)	40	55	5	1.266	56.99 cm

Extrapolated reflector thickness 20.00 cm

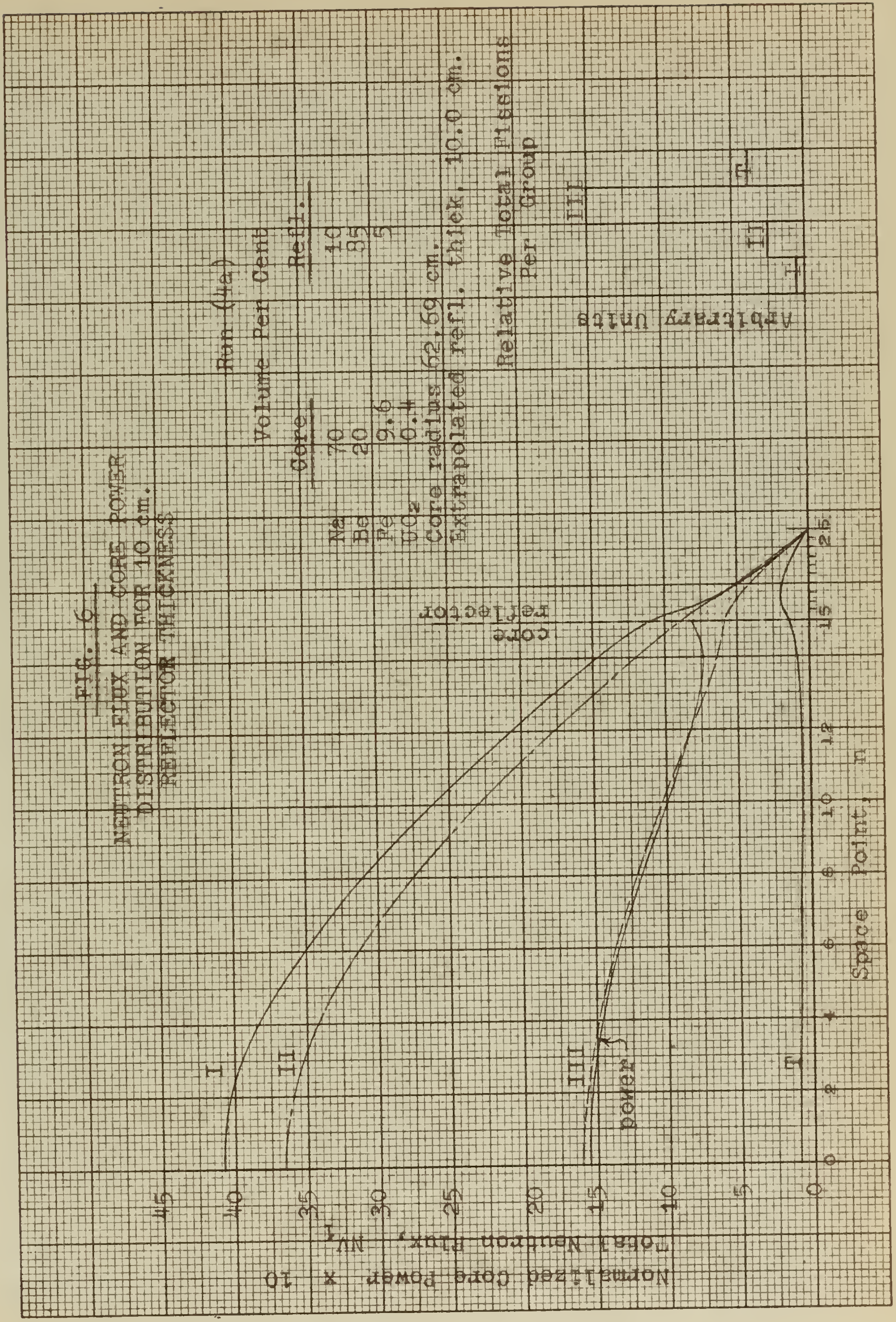
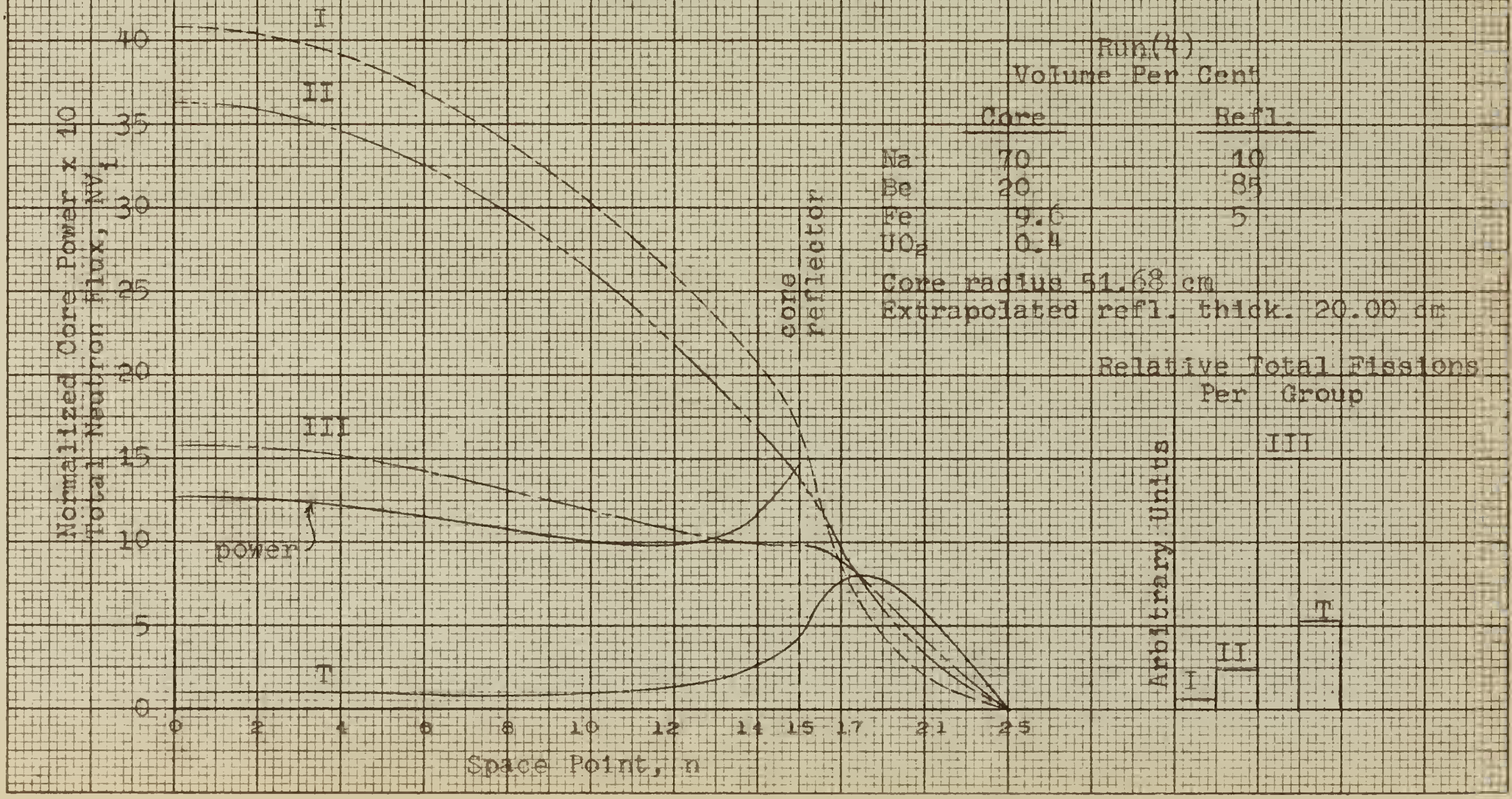


FIG. 6

NEUTRON FLUX AND CORE POWER DISTRIBUTION FOR 10 cm. REFLECTOR THICKNESS

FIG. 7

NEUTRON FLUX AND CORE POWER DISTRIBUTION FOR 20 cm. REFLECTOR THICKNESS



Run(4)

Volume Per Cent

	Core	Refl.
Na	70	10
Be	20	85
Fe	9.6	5
UO_2	0.4	

Core radius 51.68 cm

Extrapolated refl. thick. 20.00 cm

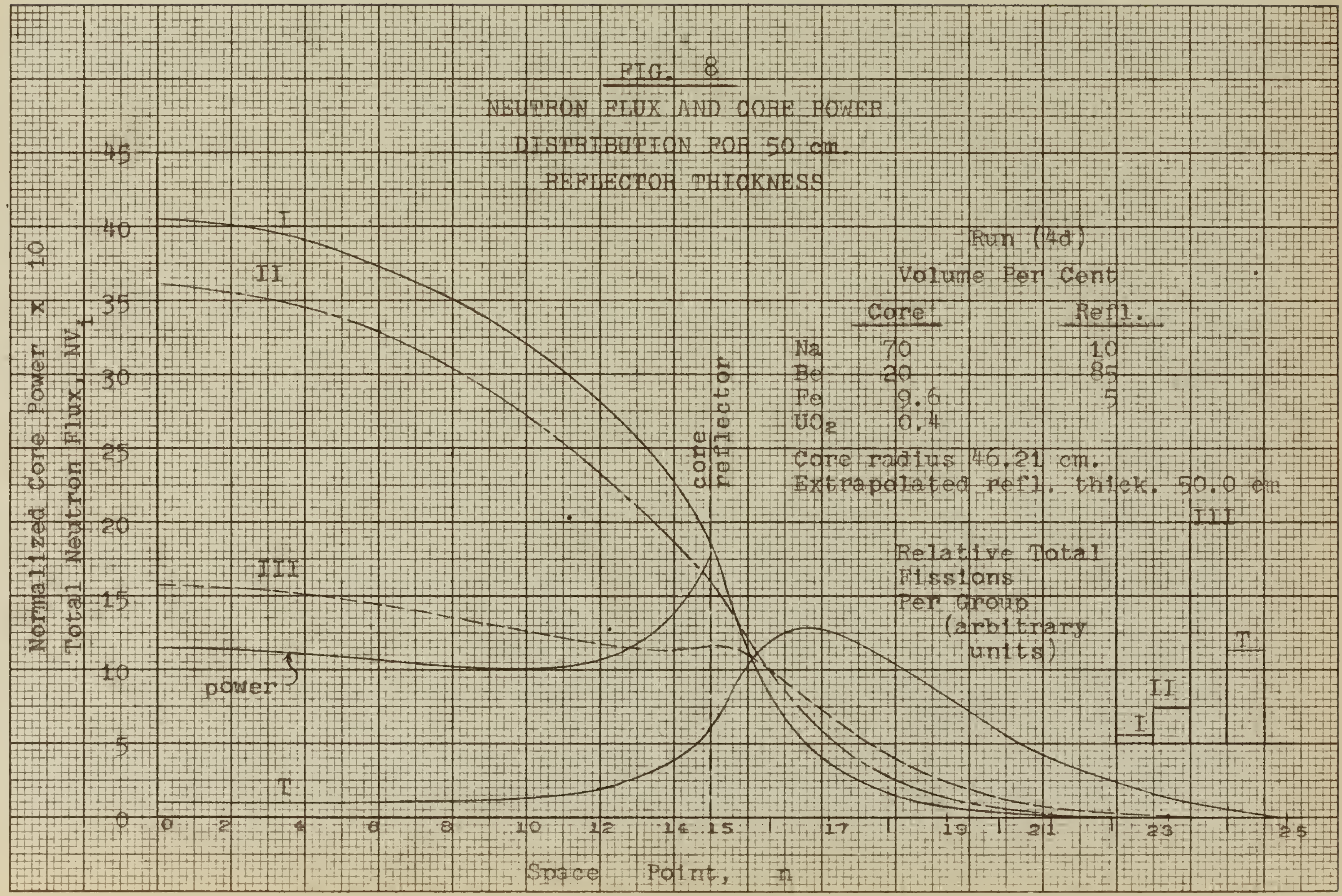


FIG. 8

NEUTRON FLUX AND CORE POWER DISTRIBUTION FOR 50 cm. REFLECTOR THICKNESS

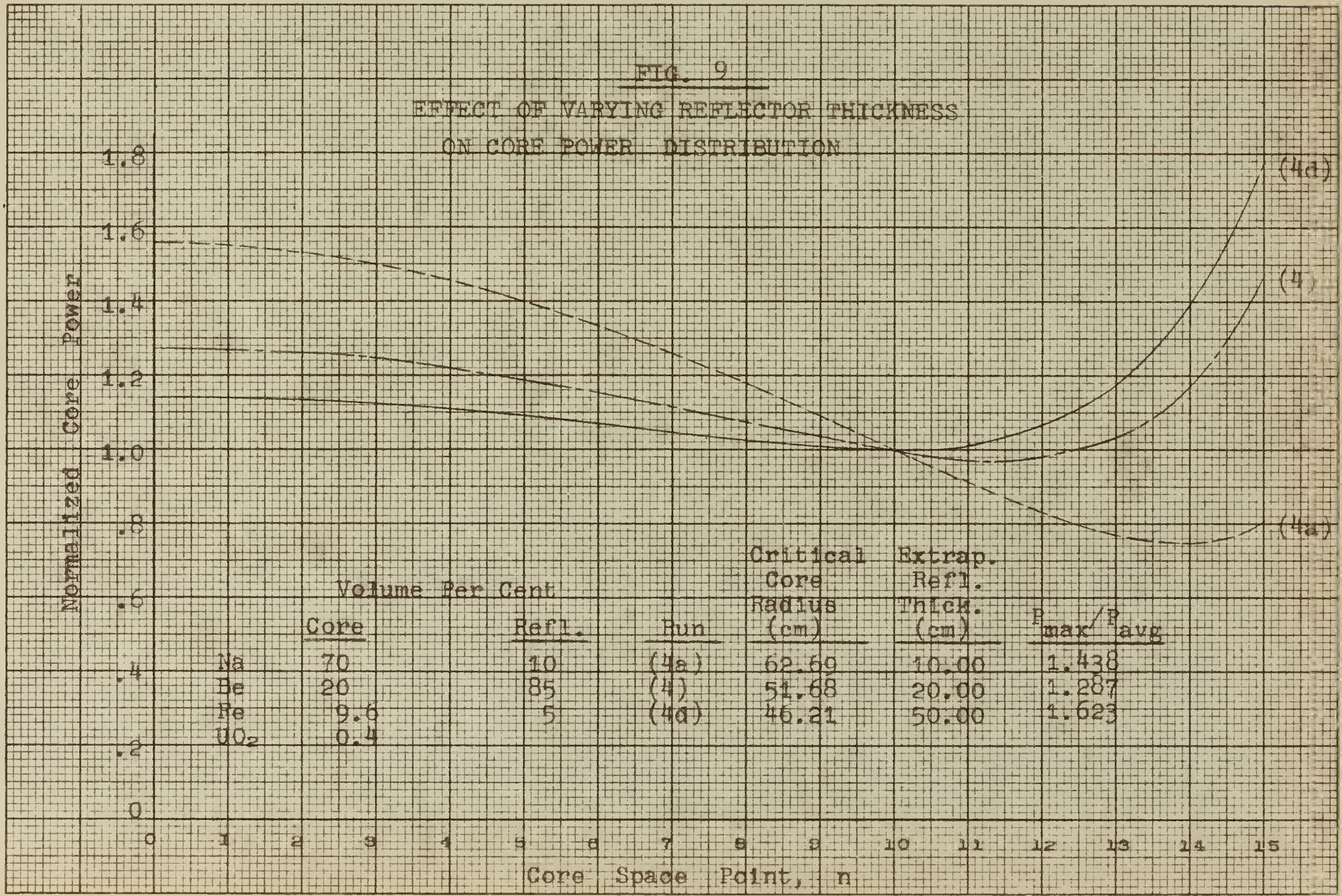


FIG. 9

EFFECT OF VARYING REFLECTOR THICKNESS ON CORE POWER DISTRIBUTION

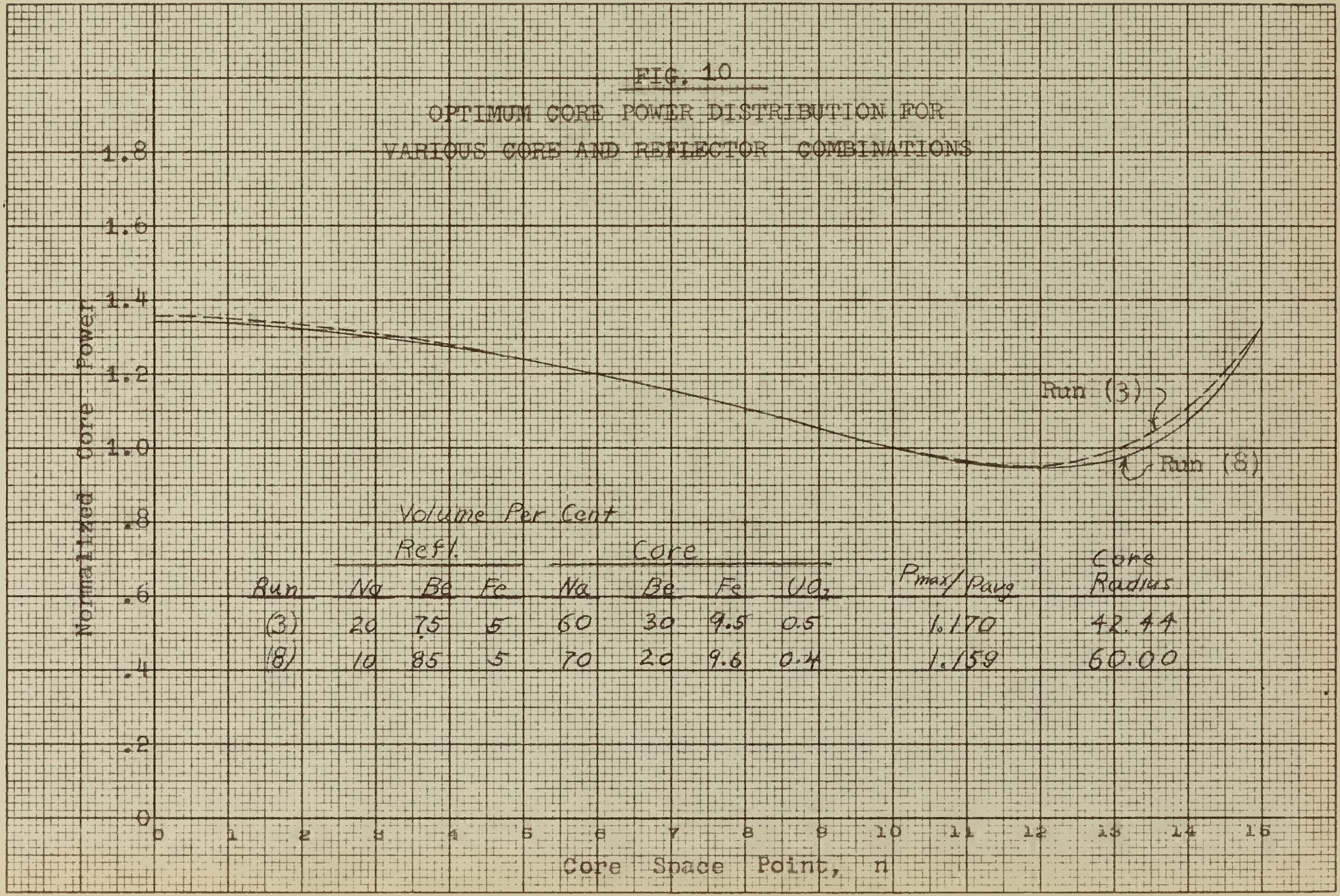
Volume Per Cent

	Core	Refl.
Na	70	10
Be	20	85
Fe	9.6	5
UO_2	0.4	

Run	Critical Core Radius (cm)	Extrap. Refl. Thick. (cm)	P_{max}/P_{avg}
(4a)	62.69	10.00	1.438
(4)	51.68	20.00	1.287
(4d)	46.21	50.00	1.623

FIG. 10

OPTIMUM CORE POWER DISTRIBUTION FOR
VARIOUS CORE AND REFLECTOR COMBINATIONS

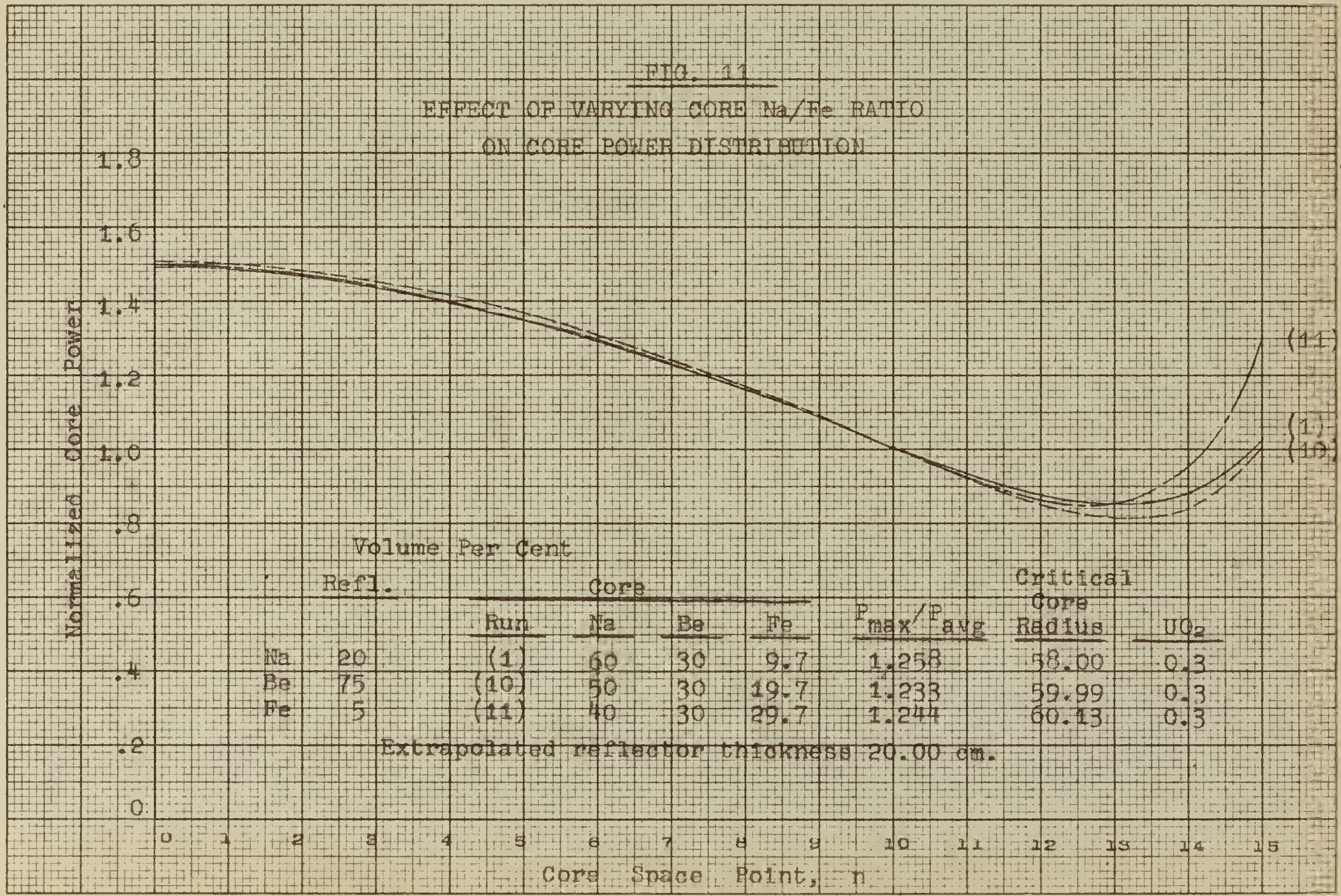


Volume Per Cent

Run	Refl. Na	Refl. Be	Refl. Fe	Core Na	Core Be	Core Fe	Core UO_2	P_{max}/P_{avg}	Core Radius
(3)	20	75	5	60	30	9.5	0.5	1.170	42.44
(8)	10	85	5	70	20	9.6	0.4	1.159	60.00

FIG. 11

EFFECT OF VARYING CORE Na/Fe RATIO
ON CORE POWER DISTRIBUTION



Volume Per Cent

Refl.	
Na	20
Be	75
Fe	5

	Core					
Run	Na	Be	Fe	P_{max}/P_{avg}	Critical Core Radius	UO_2
(1)	60	30	9.7	1.258	58.00	0.3
(10)	50	30	19.7	1.233	59.99	0.3
(11)	40	30	29.7	1.244	60.13	0.3

Extrapolated reflector thickness 20.00 cm.

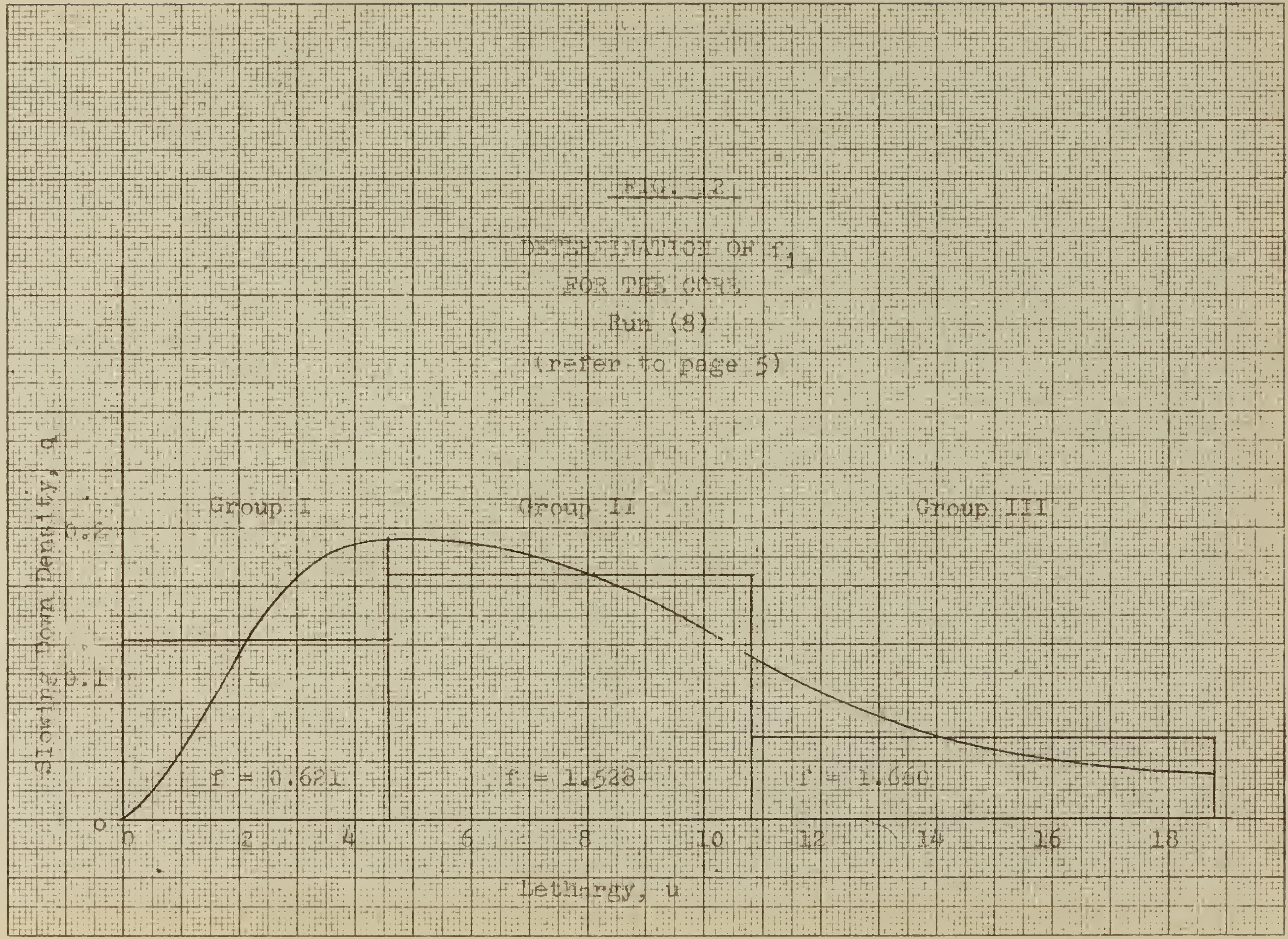


FIG. 12

DETERMINATION OF f_i

FOR THE CORE,

Run (8)

(refer to page 5)

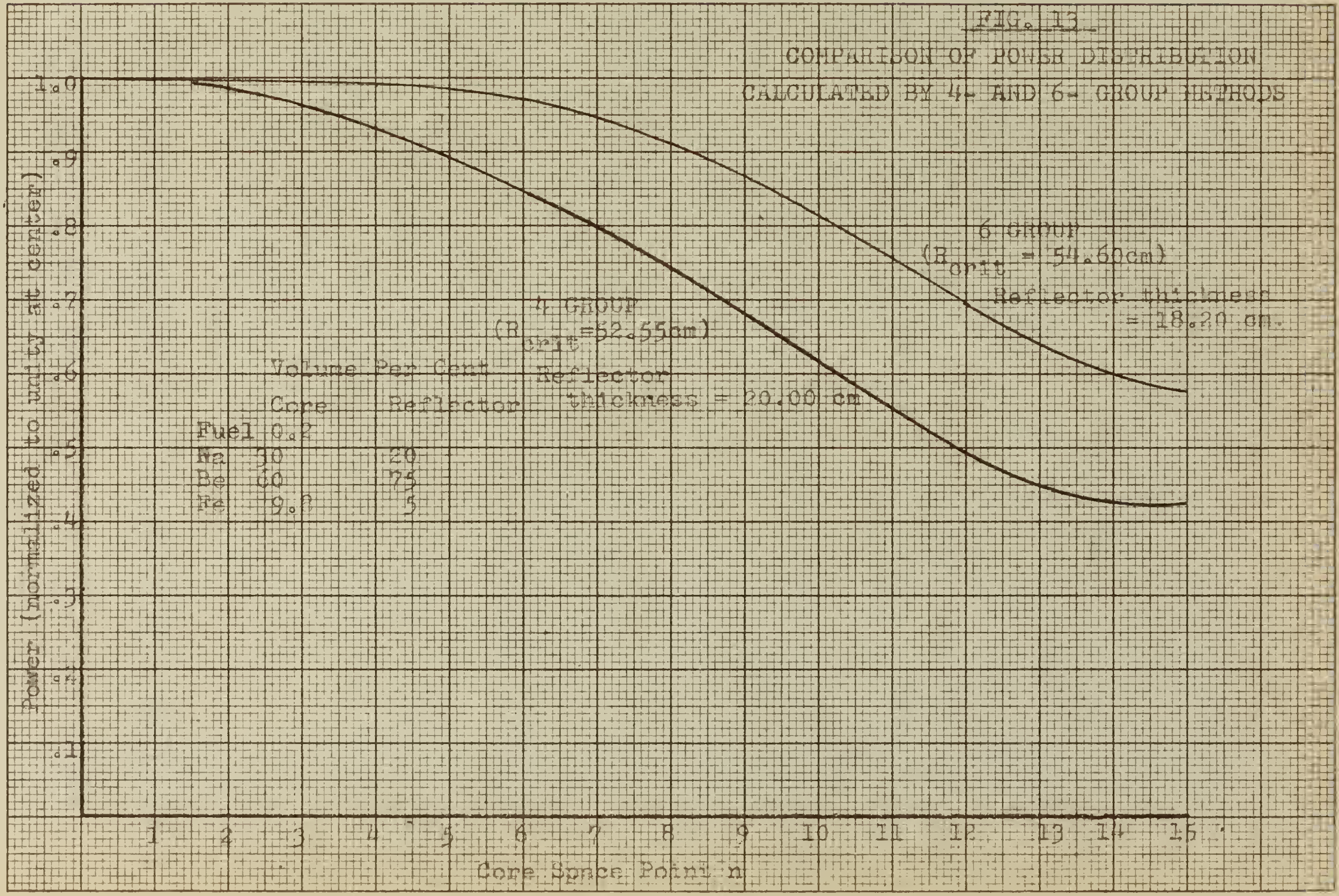


FIG. 13

COMPARISON OF POWER DISTRIBUTION
CALCULATED BY 4- AND 6- GROUP METHODS

APPENDIX A

For a typical power reactor, Glasstone[1] gives an estimated volumetric power output for the core region of 600 watts/cm.^{3}. It will be of interest to determine the approximate power potential of the reactor of Run 8, which gave optimum power distribution with the maximum power points equal at the core edge and center. The critical core radius was 60.00 cm. with sodium comprising 70 per cent of the core volume.

Because of time limitations, detailed heat transfer calculations are not made. However, an estimate of the power potential of the reactor can be made based upon the following calculations:

Core radius = 60.00 cm. = 1.97 ft.

Core volume = 9.05×10^5 cm^3

Total power output at an assumed 600 watts/cm^3

= 543 MW

It will be assumed that the sodium coolant speed is 5 ft./sec. For a mean temperature of 800°F, the sodium has the following properties[2]:

ρ = 52 lb/ft^3

c = 0.31 Btu/lb°F

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[1]--S. Glasstone, "Principles of Nuclear Reactor Engineering," D. Van Nostrand Co., Inc., New York, 1956, page 636.

[2]--"Reactor Handbook: Engineering," U. S. Atomic Energy Commission, August, 1955, (Geneva), page 266.

For power applications, it is hypothesized that a cylindrical reactor of 1.97 ft. radius can be chosen which will have a similar power distribution and volume to the spherical reactor. For 543 MW power output, the mean temperature rise, ΔT, of the coolant flowing axially through 70% of the reactor cross section is:

$$T = \frac{543 \text{ MW} \times 3.413 \times 10^{6} \frac{\text{Btu}}{\text{hr MW}}}{5 \frac{\text{ft.}}{\text{sec}} \times 3600 \frac{\text{sec}}{\text{hr.}} \times 52 \frac{\text{lb.}}{\text{ft}^{3}} \times 0.31 \frac{\text{Btu}}{\text{lb}^{\circ}\text{F}} \times 0.7 \times (1.97)^{2}\text{ft}^{2}}$$

$$= 546^{\circ}\text{F}$$

Thus a power output of 543 MW can be obtained from this reactor if sufficient heat transfer surface exists to give a mean temperature rise of 546°F with a sodium coolant velocity of 5 ft/sec. With an inlet temperature of 600°F and a mean outlet temperature of 1150°F, the maximum temperature of a thin fuel element should not be excessive.

The ability to transfer the required heat from the fuel elements to the coolant is the primary limiting factor which must be studied in determining the maximum power output.

It is of interest to compare the output of this reactor with the power requirements of a B-52 type jet bomber. At 563 miles per hour, eight jets of 10,000 lbs. thrust each develop a total of 120,000 thrust horsepower. If an overall efficiency of 20% is assumed, this is equivalent to a heat source developing 450 MW. This can be compared to the roughly estimated reactor power output of 543 MW.

APPENDIX B

TABLE I

MICROSCOPIC CROSS SECTIONS*

A. Scattering Cross Sections, σ_s (barns)

Energy E (Mev)	Lethargy, u= ln(10/E)	Na	Fe	Be	U-235	O
10	0	2.4	3.0	1.7	6.55	1.25
5.62	0.576	2.4	3.6	1.8	6.45	1.5
3.16	1.151	2.4	3.3	2.1	6.45	1.5
1.778	1.727	2.5	2.9	2.5	5.24	1.7
1	2.303	3.5	2.8	2.9	5.08	4.0
0.562	2.879	4.0	3.0	3.5	5.78	3.5
0.316	3.454	3.2	3.0	4.0	7.24	4.0
0.1778	4.030	4.0	3.5	4.9	8.7	3.5
0.1	4.606	3.2	4.0	5.7	9.76	3.5
60.01×10^{-3}	5.123	4.7	3.6		10.2	3.6
36.21 "	5.640	4.1	6.4	5.9	10.7	3.7
20.90 "	6.157	4.2	2.4		10.9	3.8
12.70 "	6.674	4.9	4.7	5.9	10.9	3.8
7.59 "	7.191	7.7	8.8		11.1	3.8
4.55 "	7.708	17.5	6.2	6.0	10.7	3.8
2.69 "	8.225	200.0	6.8		10.9	3.8
1.65 "	8.742	10.0	7.8	6.0	11.7	3.8
0.955 "	9.259	4.5	9.3		14.6	3.8
0.569 "	9.776	3.2	10.5	6.0	18.2	3.8
0.346 "	10.293	3.1	11.0		20.6	3.8
0.200 "	10.810	3.1	11.4	6.0	19.2	3.8
55.0×10^{-6}	12.142	3.1	11.4	6.0	17.6	3.8
14.45 "	13.474	3.1	11.4	6.0	19.0	3.8
4.07 "	14.806	3.2	11.4	6.0	19.0	3.8
1.00 "	16.138	3.2	11.4	6.0	3.6	3.8
0.269 "	17.470	3.2	11.4	5.9	2.5	3.8
0.070 "	18.800	3.2	11.4	5.9	10.0	3.8

---

* Neutron Cross Sections, BNL-325, U.S. Atomic Energy Commission, Aug. 1955 (Geneva Supplement)

MICROSCOPIC CROSS SECTIONS (Continued)

B. Absorption and Fission Cross Sections, σ_a and σ_f (barns)

Energy E (Mev)	Lethargy, u= ln(10/E)	Na	Fe	Be	U-235 σ_f	U-235 σ_a	O
10	0	0	0	0	1.29	1.55	0
5.62	0.576				1.29	1.55	
3.16	1.151	0	0	0	1.29	1.55	0
1.778	1.727				1.30	1.56	
1.000	2.303	0	0	0	1.27	1.52	0
0.562	2.879				1.20	1.44	
0.316	3.454	0	0	0	1.30	1.56	0
0.1778	4.030				1.50	1.80	
0.1	4.606	0	0	0	1.70	2.04	0
60.01 x 10^{-3}	5.123				2.00	2.40	
36.21 "	5.640	0	0	0	2.35	2.82	0
20.90 "	6.157				2.80	3.36	
12.70 "	6.674	0	0	0	3.4	4.08	0
7.59 "	7.191				4.1	4.92	
4.55 "	7.708	0	0	0	5.0	6.0	0
2.69 "	8.225				6.0	7.2	
1.65 "	8.742	0	0	0	7.4	8.9	0
0.955 "	9.259				9.1	10.9	
0.569 "	9.776	0	0	0	11.5	13.8	0
0.346 "	10.293				14.5	17.4	
0.200 "	10.810	0	0	0	19.0	22.8	0
55.0 x 10^{-6}	12.142				52.0	62.4	
14.45 "	13.474	0	0	0	55.0	66.0	0
4.07 "	14.806				30.0	36.0	
1.00 "	16.138	0.1	0.1	0	72.0	86.4	0
0.269 "	17.470	0.15	0.1	0	150.0	187.5	
0.070 "	18.800	0.30	0.6	0.006	320	375	0.0001

TABLE II

AVERAGE MICROSCOPIC CROSS SECTIONS*

PER ENERGY GROUP

A. Four Energy Groups (cross sections in barns)

Group I (Lethargy range 0 to 4.606)

	σ_s	σ_{tr}	σ_a	σ_f	ν	ξ
Na	3.10	3.01	0	0		0.0845
Be	3.175	2.94	0	0		0.209
Fe	3.20	3.16	0	0		0.0359
O	2.76	2.64	0	0		0.120
U-5	6.64	6.64	1.597	1.331	2.46	0.0085

Group II (Lethargy range 4.606 to 10.81)

	σ_s	σ_{tr}	σ_a	σ_f	ν	ξ
Na	22.22	21.6	0	0		0.0845
Be	5.94	5.50	0	0		0.209
Fe	7.10	7.01	0	0		0.0359
O	3.76	3.60	0	0		0.120
U-5	12.90	12.90	7.85	6.54	2.46	0.0085

Group III (Lethargy range 10.81 to 18.80)

	σ_s	σ_{tr}	σ_a	σ_f	ν	ξ
Na	3.16	3.07	0.067	0		0.0845
Be	5.97	5.54	0	0		0.209
Fe	11.4	11.26	0.083	0		0.0359
O	3.8	3.64	0	0		0.120
U-5	13.0	13.0	106.2	88.1	2.46	

Group T (Lethargy 18.80, E = 0.070 ev, 1000 F thermal energy)

	σ_s	σ_{tr}	σ_a	σ_f	ν	ξ
Na	3.2	3.11	0.30	0		0.0845
Be	5.9	5.46	0.006	0		0.209
Fe	11.4	11.26	0.60	0		0.0359
U-5	10.0	10.0	375.	320.	2.46	0.0085

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* Averaged with respect to lethargy.
U-5 stands for U_{235}

CROSS SECTIONS (Continued)

B. Six Energy Groups

6(fast): 0.234 Mev to 10 Mev
5 : 5.5 kev to 0.234 Mev
4 : 128 ev to 5.5 kev
3 : 3 ev to 128 ev
2 : .07 ev to 3 ev
1(th) : .07 ev

Na atomic density ($\times 10^{24}$): .0254 $\bar{\mu}_0$: .029 ξ : .0845
Be : .12 : .074 : .209
Fe : .0848 : .0119 : .0359
O (in UO_2) : .0494 : .0416 : .12
U-235 (90%, in UO_2) : .02224 : .00283 : .0085
U-238 : .00251 : .0028 : .0084

Absorption Cross-Sections (barns)(includes fission absorption)

Energy Group	Na	Be	Fe	O	U-235	U-238
1	0.3	0.0	0.6	0.0	374.0	1.6
2	0.1	0.0	0.2	0.0	155.0	0.8
3	0.0	0.0	0.0	0.0	54.8	0.5
4	0.0	0.0	0.0	0.0	12.4	0.5
5	0.0	0.0	0.0	0.0	3.1	0.5
6	0.0	0.0	0.0	0.0	1.6	0.5

Elastic Scattering Cross-Sections (barns):

1	3.3	5.9	11.4	3.9	11.0	8.4
2	3.3	5.9	11.4	3.8	10.5	9.8
3	3.1	6.0	11.4	3.8	8.5	9.0
4	27.4	6.0	6.4	3.8	8.3	9.0
5	3.6	5.1	4.3	3.5	8.0	9.0
6	2.7	2.2	2.7	2.6	[illegible]	9.0

Fission Cross-Sections (barns):

1	320.0
2	129.2
3	45.7
4	10.4
5	2.6
6	1.3

CROSS SECTIONS (Continued)

Since BNL-325 lists total cross sections for elements other than fissionable materials certain assumptions are necessary in order to compute scattering and absorption cross sections. The following assumptions were made with respect to all elements other than uranium:

(1) the thermal absorption cross sections listed in BNL-325 for 2200 m/s neutrons was decreased by a 1/v relation to the .07 ev thermal energy considered here;

(2) the scattering cross section was assumed to be the total cross section at the point where the absorption cross section, scaled by 1/v became negligible.

These assumptions resulted in absorption in only the thermal and next higher groups.

For the uranium the fission cross section is listed in BNL-325. The absorption cross section was synthesized from this under the assumption that the capture to fission ratio is constant. The scattering cross section of U-235, and all cross sections of U-238, proved to be unimportant for this problem. The capture to fission ratio of U-235 was taken to be 0.184.

TABLE III

REACTOR COMPOSITIONS

	Volume Per Cent						
	Core				Reflector		
Run No.	Na	Be	Fe	UO_2	Na	Be	Fe
1	60	30	9.7	0.3	20	75	5
2	60	30	9.6	0.4	20	75	5
3	60	30	9.5	0.5	20	75	5
4	70	20	9.6	0.4	10	85	5
5	70	20	9.6	0.4	20	75	5
6	70	20	9.6	0.4	40	55	5
7	60	30	9.7	0.3	10	85	5
8	70	20	9.7	0.3	10	85	5
9	80	10	9.7	0.3	10	85	5
10	50	30	19.7	0.3	20	75	5
11	40	30	29.7	0.3	20	75	5
12	75	15	9.8	0.2	5	90	5
13	70	20	9.6	0.4	15	80	5
14	60	30	9.8	0.2	20	75	5
15	60	30	9.8	0.2	40	55	5
16	30	60	9.8	0.2	20	75	5
17	30	60	9.8	0.2	40	55	5
18	90	0	9.7	0.3	25	70	5
19	60	30	9.7	0.3	2.5	95	2.5

TABLE IV

WHIRLWIND INPUT DATA
FOR FOUR ENERGY GROUPS

Run No.	Energy Group	D	f	$\xi\Sigma_s$	Σ_a	$\nu\Sigma_f$
1(C)*	I	1.867	0.756	0.02825	0.000106	0.000219
	II	0.5686	1.412	0.07548	0.000523	0.00107
	III	0.9769	1.44	0.05243	0.008781	0.01444
	T	0.9868	1.00	0.5196	0.03471	0.05247
1(R)	I	1.138	0.68	0.06154	0.0	
	II	0.531	1.08	0.12218	0.0	
	III	0.593	1.05	0.11539	0.000692	
	T	0.601	1.00	0.10581	0.00566	
2(C)	I	1.871	0.50	0.02819	0.0001412	0.000291
	II	0.569	1.75	0.07540	0.000697	0.00143
	III	0.983	1.28	0.05233	0.011136	0.01929
	T	0.990	1.00	0.05186	0.04298	0.06989
2(R)	I	1.138	0.56	0.06154	0.0	
	II	0.526	1.36	0.12236	0.0	
	III	0.593	1.18	0.11539	0.000692	
	T	0.601	1.00	0.10581	0.00460	
3(C)	I	1.870	0.50	0.02827	0.0001768	0.000364
	II	0.567	2.50	0.07548	0.000871	0.001790
	III	0.982	1.57	0.05242	0.013490	0.02412
	T	0.988	1.00	0.05194	0.051228	0.08760
3(R)	I	1.138	0.56	0.06154	0.0	
	II	0.531	1.15	0.12218	0.0	
	III	o.593	1.05	0.11539	0.000692	
	T	0.601	1.00	0.10581	0.00566	
4(C)	I	2.215	0.621	0.21473	0.0001412	0.000292
	II	0.580	1.528	0.065327	0.000697	0.00143
	III	1.189	1.660	0.038105	0.011306	0.01929
	T	1.194	1.00	0.037816	0.043668	0.06989

*(C) stands for reactor core properties, (R) for reflector properties.

WHIRLWIND INPUT DATA

(continued)

Run No.	Energy Group	D	f	$\xi\Sigma_s$	Σ_a	$\nu\Sigma_f$
4R	I	1.039	0.68	0.06884	0	
	II	0.522	1.08	0.13231	0	
	III	0.537	1.05	0.12968	0.000522	
	T	0.544	1.00	0.12820	0.003914	
5C	I	2.215	0.44	0.021473	0.0001412	0.000292
	II	0.580	2.50	0.065327	0.000697	0.00143
	III	1.189	1.57	0.038105	0.011306	0.01929
	T	1.194	1.00	0.037816	0.04367	0.06989
5R	I	1.138	0.57	0.06154	0	
	II	0.531	2.50	0.12218	0	
	III	0.593	2.00	0.11539	0.000692	
	T	0.601	1.00	0.10581	0.00566	
6C	I	2.215	0.5	0.021473	0.0001412	0.000292
	II	0.580	2.0	0.065327	0.000697	0.00143
	III	1.189	1.57	0.038105	0.011306	0.01929
	T	1.194	1.00	0.037816	0.043668	0.06989
6R	I	1.4004	0.85	0.04694	0	
	II	0.5445	1.22	0.10209	0	
	III	0.7501	1.02	0.0868	0.001033	
	T	0.7581	1.00	0.08587	0.006	
7C	I	1.867	0.621	0.02825	0.000106	0.000219
	II	0.568	1.528	0.07548	0.000523	0.00107
	III	0.9797	1.660	0.05243	0.008781	0.01444
	T	0.9868	1.00	0.05196	0.03471	0.05247
7R	I	1.034	0.68	0.06944	0	
	II	0.520	1.08	0.1331	0	
	III	0.535	1.05	0.1331	0.000491	
	T	0.540	1.00	0.1293	0.003773	
8C	I	2.209	0.621	0.02158	0.000106	0.000219
	II	0.5796	1.528	0.06535	0.0005231	0.00107
	III	1.184	1.660	0.03813	0.005871	0.01444
	T	1.190	1.00	0.03784	0.0354	0.05247
8R	I	1.039	0.68	0.06884	0	
	II	0.522	1.08	0.13231	0	
	III	0.537	1.05	0.12968	0.000522	
	T	0.544	1.00	0.12820	0.003914	

WHIRLWIND INPUT DATA

(continued)

Run No.	Energy Group	D	f	$\xi\Sigma_s$	Σ_a	$\nu\Sigma_f$
9C	I	2.700	0.621	0.01436	0.000106	0.000219
	II	0.591	1.528	0.05527	0.000523	0.00107
	III	1.498	1.660	0.02386	0.009121	0.01444
	T	1.500	1.00	0.02373	0.03631	0.05247
9R	I	1.034	0.68	0.06944	0	
	II	0.520	1.08	0.1331	0	
	III	0.535	1.05	0.1311	0.000491	
	T	0.540	1.00	0.1293	0.003773	
10C	I	1.698	0.9	0.02919	0.00011	0.000219
	II	0.564	1.2	0.07287	0.000523	0.001073
	III	0.779	1.4	0.05522	0.00931	0.01444
	T	0.784	1.0	0.05474	0.03904	0.05247
10R	I	1.138	1.0	0.06154	0	
	II	0.531	.95	0.12218	0	
	III	0.593	1.0	0.11539	0.000692	
	T	0.601	1.0	0.10581	0.00566	

TABLE V - SUMMARY OF RESULTS

Run No.	Effect Studied	ϕ_f/ϕ_t	P_{max}/P_{avg}	(cm) Critical Core Radius	(cm) Refl. Thick.	(kg) U-235 Crit. Mass	Per Cent Total Fissions in Energy Group			
							T	III	II	I
	UO_2 fuel conc.									
1	0.3	15.1	1.233	58.00	20	23.00	30.0	60.5	7.8	1.7
2	0.4	23.0	1.190	43.95		18.95	29.2	58.5	10.4	1.8
3′	0.5	41.0	1.270	42.44		13.82	23.3	58.2	16.40	2.0
	Refl. Comp.									
	Na Be Fe									
4	10 85 5	38.4	1.287	57.68	20	20.00	22.63	64.46	10.45	2.44
5	20 75 5	44.5	1.186	52.12		20.10	18.83	62.58	16.69	1.90
6	40 55 5	39.9	1.266	56.99		20.62	19.30	64.60	14.11	1.98
	Core Na/Be ratio									
7	2	16.9	1.209	45.66	20	21.21	32.2	58.5	7.8	1.5
8	3.5	25.3	1.159	60.00		23.26	23.7	67.1	6.9	2.3
9	8	48.0	1.618	65.00		23.90	22.5	65.5	9.4	2.6
	Core Na/Fe Ratio									
1	6.19	15.1	1.233	58.00	20	23.40				
10	2.54	16.6	1.258	59.99		23.61	30.0	60.5	7.8	1.7
11	1.35	18.1	1.244	60.128		23.72	29.2	60.3	8.6	1.9
8	Shape of	25.3	1.159	60.00	20	23.26	23.7	67.1	6.9	2.3
3	Optimum Power	41.3	1.170	42.44		13.82	29.2	58.5	10.4	1.8
4a	Variation of	38.57		62.69	10	21.38				
4	Reflector	38.40		51.68	20	20.00				
4d	Thickness	38.0		46.21	50	19.30				

APPENDIX C

Derivation of Equations

Equation (3.5d) of Chap. I was stated without proof and used as a basis for the boundary conditions of the four group method. A proof of equation (3.5d) follows:

Prove that

$$\left(-F + r\frac{dF}{dr}\right)\Bigg|^{r_N} = \frac{r_N}{2\Delta r}\left(F_{N+\Delta r} - F_{N-\Delta r}\right) - \frac{2}{3}F_N - \frac{1}{6}\left(F_{N+\Delta r} + F_{N-\Delta r}\right) \qquad (3.5d)$$

As a preliminary step it is necessary to prove the following relationship

$$\int_{r_1}^{r_2} r\phi\,dr = \sum_{n=r_1}^{r_2}\phi_n r_n\,\Delta r - \left(\frac{r_2}{2}\phi_{r_2} + \frac{r_1}{2}\phi_{r_1} + \frac{\Delta r}{6}\phi_{r_2} - \frac{\Delta r}{6}\phi_{r_1}\right)\Delta r \qquad (a)$$

Proof

Let $\phi = \phi_n + \left(\frac{r - r_n}{r_{n+1} - r_n}\right)\left(\phi_{n+1} - \phi_n\right)$

$$\int_{r_n}^{r_{n+1}} r\phi\,dr = \int_{r_n}^{r_{n+1}} dr\left[\phi_n r + \left(\frac{r^2 - r r_n}{r_{n+1} - r_n}\right)\left(\phi_{n+1} - \phi_n\right)\right]$$

$$= \phi_n\frac{r^2}{2} + \left(\frac{\frac{r^3}{3} - \frac{r^2 r_n}{2}}{r_{n+1} - r_n}\right)\left(\phi_{n+1} - \phi_n\right)\Bigg|_{r_n}^{r_{n+1}}$$

$$= \frac{\phi_n}{2}\left(r_{n+1}^2 - r_n^2\right) + \left(\frac{\phi_{n+1} - \phi_n}{r_{n+1} - r_n}\right)\left(\frac{r_{n+1}^3 - r_n^3}{3} - \frac{r_{n+1}^2 r_n - r_n^3}{2}\right)$$

$$= \left(\frac{r_{n+1} - r_n}{6}\right)\left[\phi_{n+1}\left(2r_{n+1} + r_n\right) + \phi_n\left(r_{n+1} + 2r_n\right)\right]$$

using the relationship

$$r_{n+1} - r_n = \Delta r$$

we have

$$\int_{r_1}^{r_N} r\phi\,dr = \frac{\Delta r}{6}\left\{\sum_{n=1}^{N-1}\left[\phi_{n+1}(2r_{n+1}+r_n)+\phi_n(r_{n+1}+2r_n)\right]\right\}$$

$$= \frac{\Delta r}{6}\left\{\sum_{n=1}^{N-1}\left[\phi_{n+1}(3r_{n+1}-\Delta r)+\phi_n(3r_n+\Delta r)\right]\right\}$$

$$= \sum_{n=1}^{N}\phi_n r_n \Delta r - \left(\frac{r_N}{2}\phi_N + \frac{r_1}{2}\phi_1 + \frac{\Delta r}{6}\phi_N - \frac{\Delta r}{6}\phi_1\right)\Delta r$$

Thus, equation (a) is proved. Next introduce the variable ψ and use the definition of ∇^2 in spherical coordinates

$$\left(-\psi + r\frac{d\psi}{dr}\right)\Bigg|_{r_1}^{r_N} = \int_{r_1}^{r_N} r\left(r\nabla^2\frac{\psi}{r}\right)dr = \int_{r_1}^{r_N} r(\psi'')\,dr \qquad (b)$$

Use equation (b) in (a) with ψ'' replacing ϕ,

where $\psi'' = \dfrac{\psi_{n+1} - 2\psi_n + \psi_{n-1}}{(\Delta r)^2}$. Then

$$\left(-\psi + r\frac{d\psi}{dr}\right)\Bigg|_{r_1}^{r_N} = \frac{1}{\Delta r}\sum_{n=1}^{N} r_n(\psi_{n+1} - 2\psi_n + \psi_{n-1})$$

$$- \frac{r_N}{2}\,\frac{(\psi_{N+1} - 2\psi_N + \psi_{N-1})}{\Delta r} - \frac{r_1}{2}\,\frac{(\psi_2 - 2\psi_1 + \psi_0)}{\Delta r}$$

$$- \frac{\psi_{N+1} - 2\psi_N + \psi_{N-1}}{6} + \frac{\psi_2 - 2\psi_1 + \psi_0}{6}$$

Appendix C (continued)

$$\left(-\psi + r\frac{d\psi}{dr}\right)\Bigg|_{r_1}^{r_N} = \frac{1}{\Delta r}\left\{\sum_{n=1}^{N} r_{n+1}\psi_{n+1} - \Delta r\sum_{n=1}^{N}\psi_{n+1}\right.$$

$$\left. -2\sum_{n=1}^{N} r_n\psi_n + \sum_{n=1}^{N} r_{n-1}\psi_{n-1} + \Delta r\sum_{n=1}^{N}\psi_{n-1}\right\}$$

$$-\frac{r_N}{2\Delta r}\left(\psi_{N+1} - 2\psi_N + \psi_{N-1}\right) - \frac{\psi_{N+1} - 2\psi_N + \psi_{N-1}}{6}$$

$$-\frac{r_1(\psi_2 - 2\psi_1 + \psi_0)}{2\Delta r} + \frac{(\psi_2 - 2\psi_1 + \psi_0)}{6}$$

$$= \frac{1}{\Delta r}\left\{r_{N+1}\psi_{N+1} - r_N\psi_N - r_1\phi_1 + r_0\phi_0 + \Delta r\left(-\psi_{N+1} - \psi_N\right.\right.$$

$$\left.\left. + \psi_1 + \psi_0\right)\right\} - \frac{r_N}{2\Delta r}\left(\psi_{N+1} - 2\psi_N + \psi_{N-1}\right) - \frac{\psi_{N+1} - 2\psi_N + \psi_{N-1}}{6}$$

$$-\frac{r_1(\psi_2 - 2\psi_1 + \psi_0)}{2(\Delta r)} + \frac{(\psi_2 - 2\psi_1 + \psi_0)}{6}$$

with $\Delta r = r_{n+1} - r_n$ the result becomes (continued)

$$= \frac{1}{\Delta r} r_N\left(\psi_{N+1} - \psi_N\right) - \frac{r_1\psi_1}{\Delta r} + \frac{r_0\psi_0}{\Delta r} - \psi_N + \psi_1 + \psi_0$$

$$-\frac{r_N}{2\Delta r}\left(\psi_{N+1} - 2\psi_N + \psi_{N-1}\right) - \frac{\psi_{N+1} - 2\psi_N + \psi_{N-1}}{6}$$

$$-\frac{r_1\psi_2}{2\Delta r} + \frac{r_1\psi_1}{\Delta r} - \frac{r_1\psi_0}{2\Delta r} + \frac{\psi_2 - 2\psi_1 + \psi_0}{6}$$

$$= \frac{r_N}{2\Delta r}\left(\psi_{N+1} - \psi_{N-1}\right) - \frac{2}{3}\psi_N - \frac{1}{6}\left(\psi_{N+1} + \psi_{N-1}\right) - \frac{r_1\psi_2}{2\Delta r}$$

$$+\frac{\psi_2}{6} + \frac{2}{3}\psi_1 + \frac{r_0\psi_0}{\Delta r} - \frac{r_1\psi_0}{2\Delta r} + \frac{7}{6}\psi_0$$

finally

$$\left(-\psi + r\frac{d\psi}{dr}\right)\Big|_{r_1}^{r_N} = \frac{r_N}{2\Delta r}\left(\psi_{N+1} - \psi_{N-1}\right) - \frac{2}{3}\psi_N$$

$$-\frac{1}{6}\left(\psi_{N+1} + \psi_{N-1}\right) - \frac{r_1\psi_2}{2\Delta r} + \frac{\psi_2}{6} + \frac{2}{3}\psi_1$$

since ψ_0 is to be identified with $F_0 = 0$. This is the proof of equation (3.5d) of Chap. I which is concerned only with the first part of the above expression relating to evaluation at the point r_N.

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